PLATE AND BOX GIRDER STIFFENER DESIGN IN VIEW OF EUROCODE 3 PART 1.5

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1. ABSTRACT

In this paper two important issues related to stability problems of stiffeners in plate and box girders are discussed: torsional stability of open stiffeners and stability of transverse stiffeners. Both problems are closely related to the new Eurocode standard EN 1993-1-5 on stability of plated structures giving answers to some open questions not solved completely in the mentioned standard.
Generally, transverse stiffeners are subjected to different loadings that tend to destabilize a stiffener. Most typical are deviation forces from longitudinal compression in the web, axial forces in the stiffener due to tension field action caused by shear forces in the web and axial forces in the stiffener from direct external loading acting on the stiffener as in the case of bearing stiffeners. Additionally to EN 1993-1-5 design rules a simplified design procedure for transverse stiffeners is proposed that includes second order effects and all relevant loadings. Based on extensive FEM simulations, criteria for preventing undesirable torsional instability of open stiffeners are also given taking into account beneficial effects of the bending stiffness of the plating and unfavourable effects of compression forces in the plating.

2. INTRODUCTION

Transverse stiffeners of plate girders are usually designed as rigid stiffeners to provide adequate shear resistance and supports for longitudinal stiffeners. Traditionally the design of rigid stiffeners is based on the stiffness criterion – minimum required stiffness for the ideal elastic case increased to take account of initial imperfections, residual stresses and post-buckling behaviour of the panels between stiffeners.

The other possibility, adopted in EN 1993-1-5 [1], is to design transverse stiffeners explicitly for strength and deformability and to consider all relevant loads acting on the stiffener, initial imperfections and second order effects. This approach is a bit more complex than the first one, but may lead to more accurate and more economical design of stiffeners.

Longitudinal stiffeners are dealt with only related to their torsional stability resistance that should be adequate to prevent premature buckling of the stiffeners and of the stiffened panel. Generally, the resistance of longitudinal stiffeners is incorporated in the resistance of longitudinally stiffened panels and the paper does not address this topic. More details on the stiffener design can be found in the commentary to the EN 1993-1-5 [2], prepared by the members of the project team that drafted the standard (the document is available free of charge on the following web page: eurocodes.jrc.it). For more reading see also [3] and [4].

3. TRANSVERSE STIFFENERS

3.1 MINIMUM REQUIREMENTS FOR TRANSVERSE STIFFENERS

According to EN 1993-1-5 transverse stiffeners should preferably provide rigid support for a plate with or without longitudinal stiffeners. They should be able to carry deviation forces from the adjacent compressed panels and be designed both for appropriate strength and stiffness.

In principle, based on the second order elastic analysis, the following criteria should be satisfied:

- maximum stress in the stiffener under design load should not exceed f_y/γ_{M1}
- additional deflection should not exceed b/300

$$\sigma_{max} \leq \frac{f_y}{\gamma_{M1}}, \quad w \leq \frac{b}{300} \qquad (1)$$

where b is the plate width (see Fig. 1), f_y is yield stress of the stiffener and γ_{M1} is partial resistance factor for the stability case.

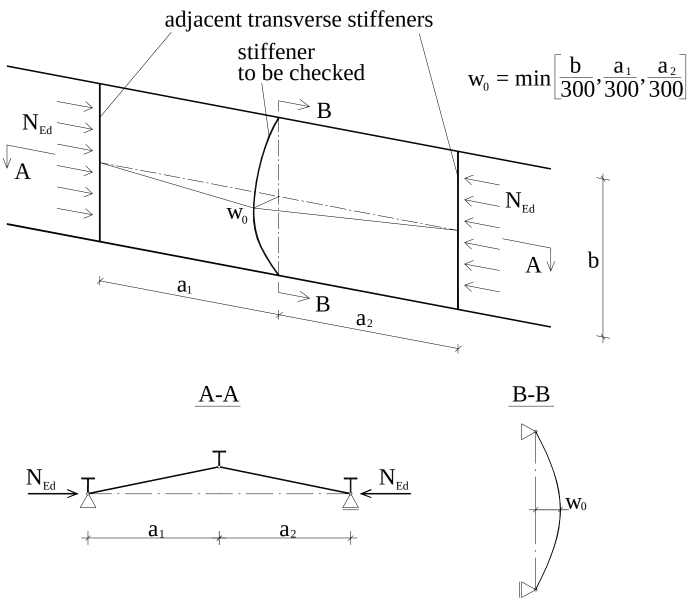


Fig. 1: Static scheme for transverse stiffener

Any other relevant load acting on the stiffener (axial force in the stiffener – e.g. due to directly applied external force – or possible horizontal transverse loading on the stiffener –

e.g. due to in-plane curvature of the girder) should be included. Also eccentricities of a stiffener should be accounted for in the presence of axial forces in the stiffener.

To check specific requirements for stiffeners an equivalent cross-section consisting of the gross cross-section of the stiffeners plus a contributing width of the plate equal to 15εt on each side of the stiffener may be used (Fig. 2), where t is the plate thickness and $\varepsilon = \sqrt{235\text{MPa} / f_y}$.

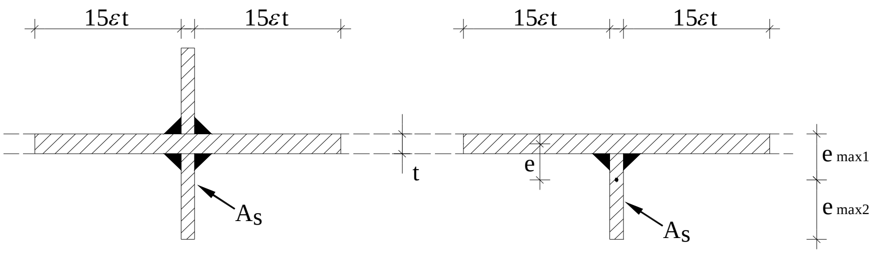


Fig. 2: Effective cross-sections of stiffeners

The static scheme that should be used to check each individual transverse stiffener is given in Fig. 1. The transverse stiffener under consideration should be treated as simply supported beam with initial sinusoidal imperfection of amplitude w_0 (Fig. 1). The adjacent compressed panels and the longitudinal stiffeners, if any, are considered to be simply supported at the transverse stiffeners, and both adjacent transverse stiffeners are supposed to be straight and rigid.
In the most general case (Fig. 3) a transverse stiffener may be loaded with:
- transverse deviation force q_{dev}, originated from longitudinal compressive force of the adjacent panels N_{Ed}
- external transverse loading q_{Ed} in the horizontal direction
- axial force in the transverse stiffener $N_{st,Ed}$, coming from vertical transverse loading on the girder
- axial force $N_{st,ten}$, originated from diagonal tension field, developed in shear. It has to be mentioned that the expression for calculating this axial force given in EN 1993-1-5 is rather conservative (due to requests from some CEN member countries).

To make the design of transverse stiffeners easier for typical cases, the requirements (1) can be transformed into a more suitable and explicit form. The most unfavourable uniform distribution of compressive force N_{Ed} along the width of the adjacent panels will be considered and other distributions will be discussed later.

The deviation force q_{dev} can be expressed as:

$$q_{dev}(x) = \left(\bar{w}_0(x) + \bar{w}(x)\right)\sigma_m = \bar{f}_0(x)\frac{N_{Ed}}{b}\left(\frac{1}{a_1}+\frac{1}{a_2}\right)\frac{\sigma_{cr,c}}{\sigma_{cr,p}} \qquad (2)$$

$$\sigma_m = \frac{\sigma_{cr,c}}{\sigma_{cr,p}}\frac{N_{Ed}}{b}\left(\frac{1}{a_1}+\frac{1}{a_2}\right) \qquad (3)$$

$$\bar{w}_0(x) = w_0 \sin\left(\frac{\pi x}{b}\right), \qquad (4)$$

where $\sigma_{cr,c}$ and $\sigma_{cr,p}$ are elastic critical column-like buckling stress and elastic critical plate-like buckling stress of the adjacent panels. Ratio $\sigma_{cr,c}/\sigma_{cr,p}$ is introduced into q_{dev} to account for the influence of plate-like behaviour of the adjacent panels that reduce deviation forces. According to EN 1993-1-5 the relevant values of the ratio $\sigma_{cr,c}/\sigma_{cr,p}$ are between 0,5 and 1,0. It is to be understood that the same limits apply for the expression (2), although this is not explicitly said in the code. For panels with large aspect ratios the ratio $\sigma_{cr,c}/\sigma_{cr,p}$ can be very small, but the values below 0,5 are not reasonable because they lead to unacceptable reduction of the deviation force q_{dev}. As a conservative solution $\sigma_{cr,c}/\sigma_{cr,p}$ may be taken as 1,0.

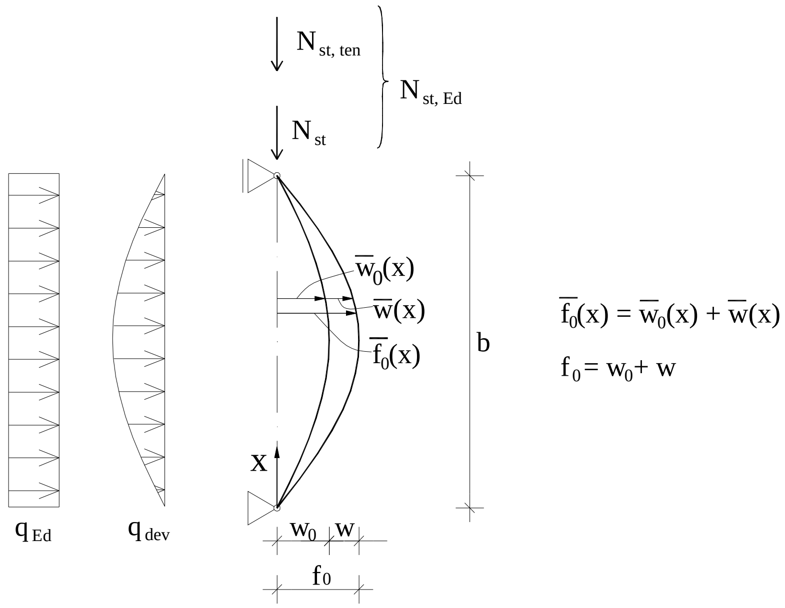


Fig. 3: General loading conditions for the transverse stiffener

When compressive force N_{Ed} is not constant along the height of the panel, as it is usually the case in plate girders, the resultant axial force from the part of the panel in compression is relevant, but taken as uniformly distributed over the height of the panel (for the sake of simplicity and to be on the safe side). In order to maintain the importance of requirements (1) for transverse stiffeners, the compressive force N_{Ed} should not be taken less than the largest compressive stress times half the effective compression area of the panel including longitudinal stiffeners. This limitation may be decisive for instance for symmetric plate girders in pure bending. When axial forces in adjacent panels differ, the larger of the two is taken into consideration.
For the four typical cases (see Chapters 3.2 to 3.5) direct requirements will be derived from requirements (1).

3.2 STIFFENED PANELS LOADED WITH LONGITUDINAL COMPRESSION FORCES N_{ED} ONLY

When the transverse stiffener is loaded only with the deviation forces coming from longitudinal compression force in the panels N_{Ed}, the requirements (1) are presumably satisfied by providing the transverse stiffener with a minimum second moment of inertia I_{st}.

With N_{Ed} uniformly distributed over the width of the panel and $\overline{w}_0(x)$ taken as sine function from Eq. (2), both the additional deflection $\overline{w}(x)$ and the deviation force $q_{dev}(x)$ have also a sinusoidal shape. By taking this into consideration, maximum stress σ_{max} and maximum additional deflection w can be calculated as:

$$\sigma_{max} = \frac{M_{max} e_{max}}{I_{st}} = \frac{q_{dev,0} b^2 e_{max}}{\pi^2 I_{st}} = \frac{(w_0 + w)\sigma_m b^2 e_{max}}{\pi^2 I_{st}} \quad (5)$$

$$w = \frac{q_{dev,0} b^4}{\pi^4 E I_{st}} = \frac{(w_0 + w)\sigma_m b^4}{\pi^4 E I_{st}} = \frac{\sigma_{max} b^2}{\pi^2 E e_{max}} \quad (6)$$

where

M_{max} is maximum value of the bending moment in the stiffener caused by the deviation force

e_{max} is the distance from the extreme fibre of the stiffener to the centroid of the stiffener (see Fig. 2)

$q_{dev,0}$ $=(w_o+w)\sigma_m$ is the amplitude of the deviation force $q_{dev}(x)$.

By introducing (6) into (5) a relation between I_{st} and σ_m can be determined:

$$I_{st} = \frac{\sigma_m}{E}\left(\frac{b}{\pi}\right)^4 \left(1 + w_0 \frac{\pi^2 E e_{max}}{b^2 \sigma_{max}}\right) \quad (7)$$

or, with due account taken of (6), I_{st} can be expressed in terms of w

$$I_{st} = \frac{\sigma_m}{E}\left(\frac{b}{\pi}\right)^4 \left(1 + \frac{w_0}{w}\right). \quad (8)$$

To get minimum allowable values for I_{st}, maximum allowable values of $\sigma_{max}=f_y/\gamma_{M1}$ and w=b/300 are introduced in (7) and (8), respectively. Because of (6) expressions (7) and (8) can be then merged in one condition given in EN 1993-1-5:

$$I_{st} \geq \frac{\sigma_m}{E}\left(\frac{b}{\pi}\right)^4 \left(1 + w_0 \frac{300}{b} u\right) \quad (9)$$

$$u = \frac{\pi^2 E e_{max} \gamma_{M1}}{b 300 f_y} \geq 1. \quad (10)$$

When u is less than 1,0, a displacement check is decisive and u is taken as 1,0 in (9), otherwise a strength check is in force.

3.3 STIFFENED PANELS LOADED WITH LONGITUDINAL COMPRESSION FORCES N_{ED} AND AXIAL FORCES IN THE DOUBLE SIDED TRANSVERSE STIFFENER $N_{ST,ED}$ (N_{ST} AND/OR $N_{ST,TEN}$)

When in addition to the deviation forces the double sided transverse stiffener is loaded with an external axial force, then the deviation force is transformed into an additional axial force $\Delta N_{st,Ed}$ in the stiffener:

$$\Delta N_{st,Ed} = \frac{\sigma_m b^2}{\pi^2}. \quad (11)$$

The mechanical model is for this case shown in Fig. 3 (excluding q_{Ed}). The stiffener is loaded with a deviation force from longitudinal compression in the panels (N_{Ed}) and axial force $N_{st,Ed}$ in the stiffener, resulting from the tension field action ($N_{st,ten}$) and/or from the external loading (N_{st}).

Equilibrium differential equation of the stiffener can be written as:

$$EI_{st}\bar{w}_{,xxxx} + N_{st,Ed}(\bar{w}_{0,xx} + \bar{w}_{,xx}) = q_{dev}(x) = \sigma_m(\bar{w}_0 + \bar{w}) \tag{12}$$

or

$$\bar{w}_{,xxxx} + \omega^2\bar{w}_{,xx} - \alpha^2\bar{w} = \alpha^2\bar{w}_0 - \omega^2\bar{w}_{0,xx} \;,\; \omega^2 = \frac{N_{st,Ed}}{EI_{st}} \;,\; \alpha^2 = \frac{\sigma_m}{EI_{st}} \tag{13}$$

Sine function

$$\bar{w} = w \sin\frac{\pi x}{b} \tag{14}$$

automatically fulfils all static and kinematic boundary conditions and can be taken as a suitable solution of (13). A free constant *w* is easily obtained from (13):

$$w = \frac{\omega^2\frac{\pi^2}{b^2} + \alpha^2}{\frac{\pi^4}{b^4} - \omega^2\frac{\pi^2}{b^2} - \alpha^2} w_o = K w_o \tag{15}$$

Constant K can be rewritten as follows:

$$K = \frac{\beta^2}{\frac{\pi^4}{b^4} - \beta^2} \tag{16}$$

where

$$\beta^2 = \frac{\pi^2}{EI_{st}b^2}(N_{st,Ed} + \Delta N_{st,Ed}) \text{ and } \Delta N_{st,Ed} = \frac{\sigma_m b^2}{\pi^2} \tag{17}$$

By introducing

$$\Sigma N_{st,Ed} = N_{st,Ed} + \Delta N_{st,Ed} \tag{18}$$

and the Euler buckling strength of a stiffener

$$N_{cr,st} = \frac{\pi^2 EI_{st}}{b^2} \tag{19}$$

the amplitudes of the additional deflection w and total deflection f are:

$$w = w_o \frac{1}{\frac{N_{cr,st}}{\Sigma N_{st,Ed}} - 1} \tag{20}$$

$$f = w_o + w = w_o \frac{1}{1 - \frac{\Sigma N_{st,Ed}}{N_{cr,st}}}\,. \tag{21}$$

Comparing (20) and (21) to the standard solution for the compressed imperfect bar, it is evident that the deviation force q_{dev} coming from longitudinal compression in the web panels (N_{Ed}) can be replaced by the additional axial force $\Delta N_{st,Ed}=\sigma_m b^2/\pi^2$ in the stiffener. $\Delta N_{st,Ed}$ is a small fraction of a longitudinal compression force N_{Ed} in the plate panels. This solution is very simple and easy to apply. When the distribution of the longitudinal compression stresses is not constant (i.e. web panel of a girder under bending moment), the results are on the safe side.

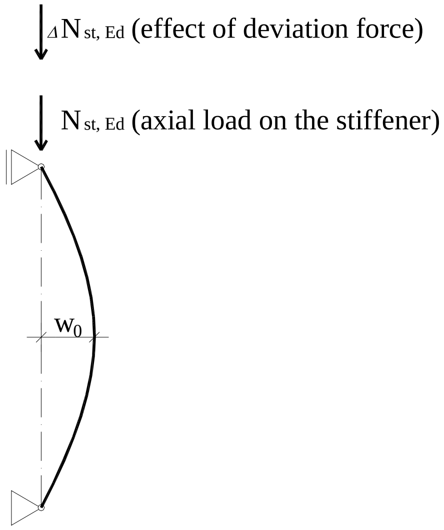


Fig. 4: Simplified analysis of the axially loaded transverse stiffener

Both stiffness and strength requirements (see (1)) may be checked according to the following procedure that takes second order effects into account:

$$w = w_o \frac{1}{\frac{N_{cr,st}}{\Sigma N_{st,Ed}} - 1} \leq \frac{b}{300} \tag{22}$$

$$\sigma_{max} = \frac{N_{st,Ed}}{A_{st}} + \frac{\Sigma N_{st,Ed} e_{max}}{I_{st}} f =$$

$$= \frac{N_{st,Ed}}{A_{st}} + \frac{\Sigma N_{st,Ed} e_{max}}{I_{st}} w_o \frac{1}{1 - \frac{\Sigma N_{st,Ed}}{N_{cr,st}}} \leq \frac{f_y}{\gamma_{M1}} \tag{23}$$

Note that only the axial force $N_{st,Ed}$ needs to be considered in the first term of (23). Rather than being a real axial force, $\Delta N_{st,Ed}$ is simply equivalent in effects to the deviation force q_{dev}. For the case $N_{st,Ed} = 0$ (see Fig. 4), (22) and (23) reduce to (9). Requirements (22) and (23) are valid only for double sided stiffeners.

3.4 STIFFENED PANELS LOADED WITH LONGITUDINAL COMPRESSION FORCES N_{ED} AND AXIAL FORCES IN THE SINGLE SIDED TRANSVERSE STIFFENER $N_{ST,ED}$ *(*N_{ST} AND/OR $N_{ST,TEN}$*)*

For single sided transverse stiffener EN 1993-1-5 does not provide any design rules and the following procedure may be used. The mechanical model of a single sided stiffener is shown in Fig. 5. The equilibrium equation (13) is still valid, only the boundary conditions change due to end moments $M_{EN} = N_{st,Ed}\, e_o$, where e_o is the eccentricity of the centroid of single sided stiffener relative to the mid plane of the web. With new boundary conditions the solution of (13) becomes much more complicated than the solution given by (20) and is not suitable for practical use. To overcome this problem, a simplified approach may be used, based on the expression for maximum displacements and stresses at mid height of double sided stiffeners (22) and (23).

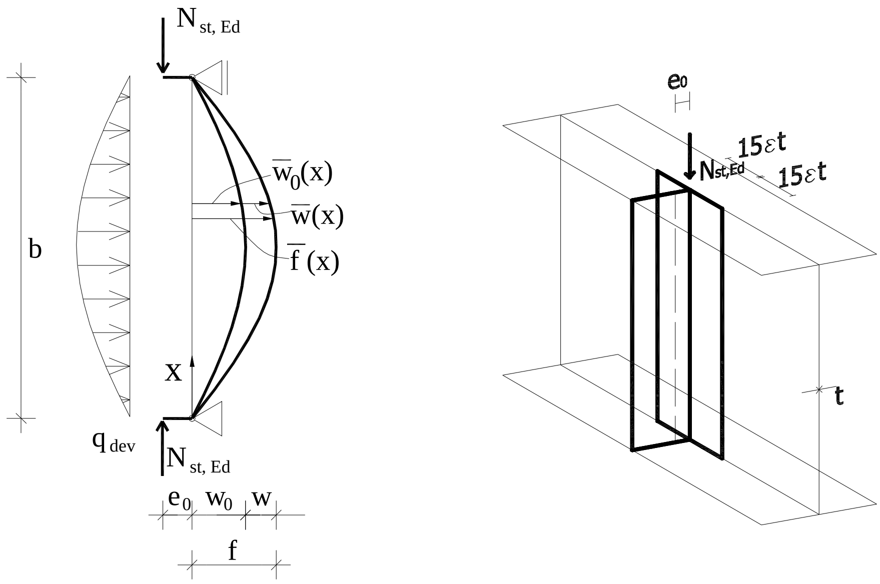


Fig. 5: Mechanical model of a single sided stiffener

In this simplification it is considered that $N_{st,Ed}$ is related to the maximum eccentricity $e_o + w_o$ and $\Delta N_{st,Ed}$ from deviation force only to w_o. In this case expression (23) rewrites as follows

$$\sigma_{max} = \frac{N_{st,Ed}}{A_{st}} + \frac{e_{max}}{I_{st}}\left(\Sigma N_{st,Ed} w_0 \cdot \frac{1}{1-\frac{\Sigma N_{st,Ed}}{N_{cr,st}}} + N_{st,Ed} e_0 \cdot \frac{1}{1-\frac{\Sigma N_{st,Ed}}{N_{cr,st}}}\right) \quad (24)$$

and after rearranging

$$\sigma_{max} = \frac{N_{st,Ed}}{A_{st}} + \frac{\Sigma N_{st,Ed} e_{max} w_0}{I_{st}} \cdot \frac{1}{1-\frac{\Sigma N_{st,Ed}}{N_{cr,st}}}\left(1+q_m\right) \le \frac{f_y}{\gamma_{M1}} \quad (25)$$

where

$$q_m = \frac{N_{st,Ed}\, e_0}{\Sigma N_{st,Ed}\, w_0} \quad (26)$$

If the same amplification factor $(1+q_m)$ is applied to the displacements, equation (22) rewrites as follows:

$$w = w_0 \frac{1}{\frac{N_{cr,st}}{\Sigma N_{st,Ed}} - 1}\left(1+q_m\right) \le \frac{b}{300} \quad (27)$$

Expressions (24) and (25) were tested against the solution of the equilibrium equation (13) and it was found [5], based on extensive parametric study, that safe and very accurate results are obtained, when in (25) q_m is multiplied by a factor 1,11 and in (27) by a factor

1,25. The results of this parametric study that covered all important parameters (2431 cases) are summarized in the Table 1.

Moments				Displacements			
total x_m	2431	total $x_m < 1$	329	total x_w	2431	total $x_m < 1$	176
x_{min}	0,982	x_{min}	0,982	x_{min}	0,989	x_{min}	0,989
x_{max}	1,103	x_{max}	0,999	x_{max}	1,016	x_{max}	0,996
x_{avg}	1,029	x_{avg}	0,990	x_{avg}	1,005	x_{avg}	0,990
std dev	0,031	std dev	0,006	std div	0,006	std div	0,001

Table 1: Results of the parametric study – reliability check

Parameters x in the Table 1 are ratios of the theoretical values of displacements or bending moments obtained from (13) and corresponding values given by simplified expressions (28) and (29). This means that single sided transverse stiffeners may be checked to fulfil the requirements (1) with the following simplified expressions:

$$\sigma_{max} = \frac{N_{st,Ed}}{A_{st}} + \frac{\Sigma N_{st,Ed} e_{max} w_0}{I_{st}} \cdot \frac{1}{1 - \frac{\Sigma N_{st,Ed}}{N_{cr,st}}} \left(1 + 1{,}11 q_m\right) \le \frac{f_y}{\gamma_{M1}} \tag{28}$$

$$w = w_0 \frac{1}{\frac{N_{cr,st}}{\Sigma N_{st,Ed}} - 1} \left(1 + 1{,}25 q_m\right) \le \frac{b}{300} \tag{29}$$

At single sided stiffeners e_{max} may be replaced with the distance from the web surface (opposite to the stiffener) to the stiffener centroid, if this distance is smaller than e_{max}. This is due to the fact that the most unfavourable situation is present when the initial bow imperfection w_o extends to the stiffener side of the web. In this case compression stresses from the axial force and from bending sum up at the web side of the stiffener.

3.5 GENERAL CASE

In the most general case, where besides deviation forces q_{dev} also transverse loading q_{Ed} and axial force $N_{st,Ed}$ act on the stiffener, deviation force q_{dev} shall be calculated explicitly and then used in the analysis of the stiffener. The numerical models for double and single sided stiffeners are shown in Fig. 6a and Fig. 6b, respectively. The deviation force q_{dev} depends on the additional deflection w(x) that depends by itself on the loads N_{Ed}, $N_{st,Ed}$, q_{Ed} acting on the stiffener. For this reason an iterative procedure is required to calculate q_{dev}.

Due to the assumed sinusoidal shape of the initial imperfections and assuming the sinusoidal shape also for the transverse loading q_{Ed} (for the sake of simplicity), the deviation force q_{dev} writes:

$$q_{dev}(x) = \sigma_m \left(w_0 + w\right) \sin\left(\frac{\pi x}{b}\right) \tag{30}$$

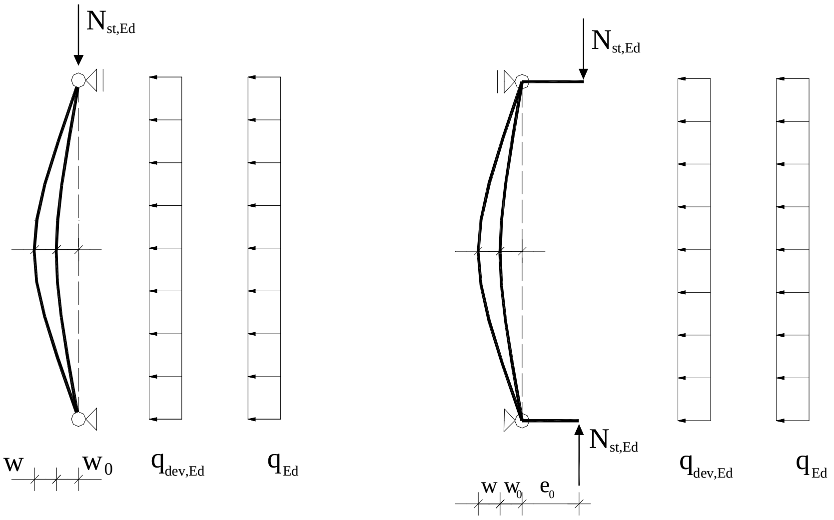
where w is the additional deflection due to a deviation force that needs to be determined iteratively (effects of the second order theory) or conservatively be taken equal to the maximum additional deflection w = b/300. With this simplification, the iterative procedure is avoided, but certainly a displacement check has to be performed to assure that $w(q_{dev}, q_{Ed}, N_{st,Ed}) \leq b/300$. In most cases this simplification does not result in any significant increase in the stiffener cross-sectional size.

Instead of the sinusoidal deviation force, EN 1993-1-5 proposes an equivalent uniformly distributed deviation force that causes the same maximum bending moment in the stiffener:

$$q_{dev,eq} = \frac{\pi}{4}\sigma_m\left(w_0 + w\right) \tag{31}$$

Actually, theoretically correct transformation parameter is $8/\pi^2$ instead of $\pi/4$, but the difference is only 3%.

In the presence of axial force $N_{st,Ed}$ the second order analysis should be performed even if w is taken as b/300.



a) double sided stiffener b) single sided stiffener

Fig. 6: Transverse stiffener under general loading conditions

This general approach is easily applicable to the first two cases where only longitudinal axial force in the adjacent panels (first case) and in addition axial force in the stiffeners (second case) are present. For the first case for instance $q_{dev,eq}$ may be calculated form (31) by taking w = b/300. From the corresponding bending moments and deflections requirements (1) may be checked.

4. TORSIONAL STABILITY OF OPEN LONGITUDINAL (AND TRANSVERSE) STIFFENERES

4.1 INTRODUCTION

Economical design of plate and box girders usually requires longitudinal stiffeners to be used in compression parts of webs and in lower flanges around internal supports of continuous girders. Stiffeners of upper flanges are strongly influenced by transverse loading acting on the girder and are not covered in this paper. Longitudinal stiffeners are usually supported by rigid transverse stiffeners and are subjected mainly to longitudinal compression stresses that may cause flexural buckling, and in the case of open cross sections also torsional buckling.

To prevent premature torsional buckling, design codes typically put requirements for minimum torsional resistance such that torsional buckling is not critical. The easiest way to fulfil this requirement in all practical cases is to limit the slenderness at torsional buckling to the length of the plateau of the buckling curve. In EN 1993-1-5 [1] that covers plated structural elements the plateau length is set to $\bar{\lambda} = 0.7$ for flat stiffeners and $\bar{\lambda} = 0.4$ for stiffeners with warping stiffness, such as L and T stiffeners (see Fig. 7).

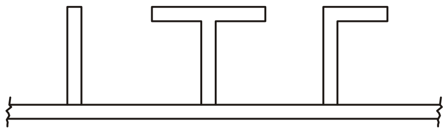


Fig. 7: Open stiffeners

The plateau length of the flat stiffeners $\lambda = 0.7$ is taken as equal as in the plate buckling problem and the plateau length of the stiffener with the warping stiffness $\lambda = 0.4$ is taken as equal as in the lateral torsional buckling problem, because lateral torsional buckling of a stiffener takes the form of these two buckling phenomena.

The two criteria in EN 1993-1-5 are written in different format, specifying the minimum value of σ_{cr}:

$$\bar{\lambda} = \sqrt{\frac{f_y}{\sigma_{cr}}} \le k, \qquad k = 0.4 \text{ or } 0.7 \tag{32}$$

$$\sigma_{cr} \ge \vartheta f_y, \quad \vartheta = \frac{1}{k^2} = 6 \text{ or } 2 \tag{33}$$

For flat stiffeners σ_{cr} should be at least two times and for other stiffeners six times larger than yield stress f_y (see Figure 8).

For the torsional buckling the following expression for σ_{cr} may be written, assuming that the axis of rotation coincides with attachment line between the stiffener and the plate [6]:

$$\sigma_{cr} = \frac{1}{I_p}\left(\frac{\pi^2 E I_w}{l^2} + G I_t\right) \tag{34}$$

where

I_p = polar radius of giratation

I_y, I_z – second moment of inertia of the stiffener cross section

I_w – warping constant

I_t – Saint-Venant torsional constant

E, G – elastic and shear modulus

l – buckling length of the stiffener (usually the distance between transverse stiffeners)

For flat stiffener (see Fig. 9) with $I_w = 0$, the criterion $\sigma_{cr} > 2\ f_y$ may be rewritten in the following form:

$$\frac{b_s}{t_s} \leq 0.43\sqrt{\frac{E}{f_y}} = \begin{cases} 13.1 & (S235) \\ 10.7 & (S355) \end{cases} \tag{35}$$

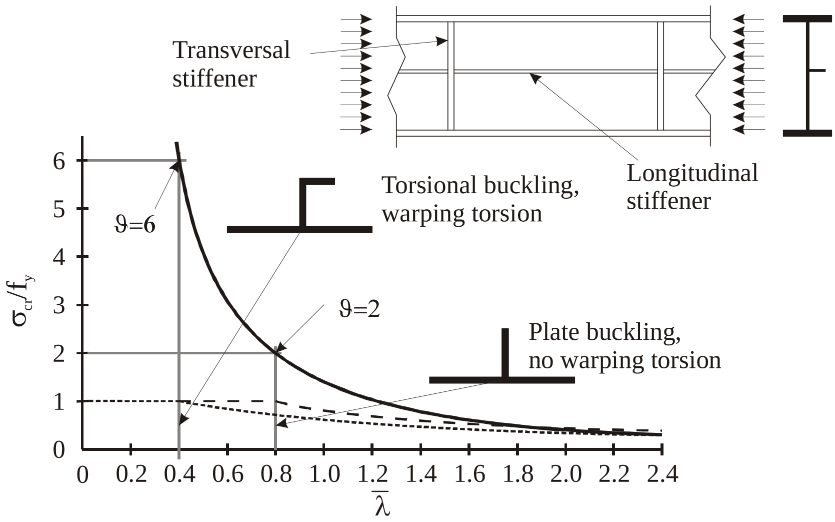


Fig. 8: Limiting values of σ_{cr} and $\bar{\lambda}$ for open stiffeners

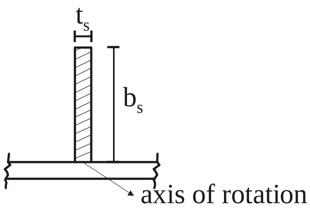


Fig. 9: Flat stiffener

In EN 1993-1-5 this criterion is given in a different form:

$$\frac{I_t}{I_p} \geq 5.3\frac{f_y}{E} \tag{36}$$

It is obvious that this criterion is easy to fulfil by keeping b_s/t_s ratio small enough (as in the plate buckling problem).

Criterion (34) is much more difficult to fulfil in the case of stiffeners with warping torsional stiffness. The reason is not so much in the higher value of σ_{cr} but in the fact that I_t is small and the I_w/I_p ratio takes over the control. Larger stiffener does not help much, as both parameters I_w and I_p increase simultaneously and sometimes σ_{cr} even becomes smaller at larger stiffener cross section.

One possibility to solve this problem is to take account of the plate that gives some continuous torsional restraint to the stiffener, as it is described in the Commentary to EN 1993-1-5 [2]. The magnitude of the torsional restraint depends on the stiffener spacing, plate slenderness and the level of longitudinal compression stresses that are present in the plate. These stresses reduce the effectiveness of the torsional restraint by the plate and the main aim of this paper is to present an attempt to assess this influence for longitudinally stiffened plates in pure compression.

4.2 CRITICAL STRESSES – TORSIONAL RESTRAINT OF THE PLATE

Equilibrium equation of the stiffener with a continuous elastic torsional support c_ϑ (see Fig. 10) may be written as

$$E \cdot I_w \cdot \vartheta_{,xxxx} + (N \cdot i_p^2 - G \cdot I_t) \cdot \vartheta_{,xx} + c_\vartheta \cdot \vartheta = 0 \quad (37)$$

or

$$\vartheta_{,xxxx} + \omega^2 \cdot \vartheta_{,xx} + \alpha^2 \cdot \vartheta = 0 \quad (38)$$

where

ϑ = torsional rotation

$$\alpha^2 = \frac{c_\vartheta}{EI_w}$$

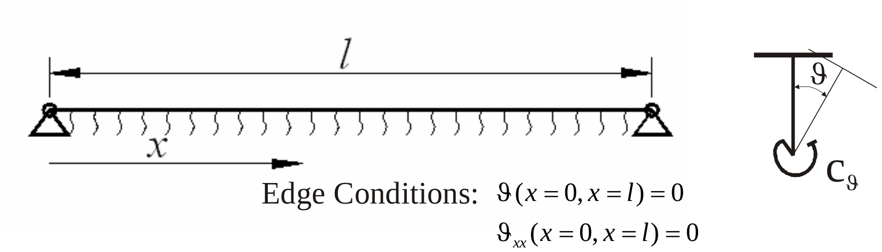


Fig. 10: Numerical model of the stiffener supported continuously by an elastic torsional support c_ϑ

The first two articles of Eq (37) correspond to the classic torsional buckling and the third article comes from the torsional restraint. The solution of Eq (37) that fulfils all boundary conditions may be given in the form of:

$$\vartheta = A \cdot \sin \frac{m \cdot \pi \cdot x}{l} \quad (39)$$

where m is an integer describing the number of half buckles. By applying (39) in Eq (38) the following solution for the critical force $\sigma_{cr} = N_{cr}/A$ is obtained:

$$\sigma_{cr} = \frac{1}{I_p}\left[E \cdot I_w \frac{m^2 \cdot \pi^2}{l^2} + c_\vartheta \frac{l^2}{m^2 \cdot \pi^2} + G \cdot I_t\right] \quad (40)$$

First derivation of Eq (40) by m gives the criterion for the minimum value of σ_{cr}

$$\frac{m^2 \cdot \pi^2}{l^2} = \sqrt{\frac{c_\vartheta}{E \cdot I_w}} \quad (41)$$

and

$$\sigma_{cr-MIN} = \frac{1}{I_p}\left[2\sqrt{c_\varphi \cdot E \cdot I_w} + G \cdot I_t\right] \quad (42)$$

From Equation (41) it is clear that for stiffener length l larger than

$$l_{lim} = \pi \sqrt[4]{\frac{E \cdot I_w}{c_\vartheta}} \quad (43)$$

buckling in more than one half buckle occurs according to Eq (42), and for stiffener lengths shorter than l_{lim} buckling in one half buckle occurs according to Eq (40) with m =

1. Initial torsional stiffness that is not affected by compression stresses may be determined by straight forward static analysis. For only one stiffener $c_{\vartheta 0}$ is given by

$$c_{\vartheta 0} = \frac{E \cdot t^3}{2 \cdot b} \tag{44}$$

and for a large number of equally spaced stiffeners by

$$c_{\vartheta 0} = \frac{E \cdot t^3}{3 \cdot b} \tag{45}$$

4.3 NUMERICAL ANALYSIS

To determine the influence of longitudinal compression stresses on the continuous elastic torsional restraint c_ϑ an extensive numerical parametric study was performed using ABAQUS computer code. The aim of the analysis was to determine the reduction of the torsional stiffness of the plate for the rotations around the stiffener-plate connection line with increasing longitudinal compression in the plate. The stiffener itself was not included in the analysis. It was assumed that the stiffener-plate connection line is laterally supported because torsional stiffness in the plate is relevant only prior to flexural buckling of the stiffener. The main parameters that were taken into account in the parametric study were: plate thickness t (8 – 32 mm), subpanel slenderness b/t (31 – 125), panel length/width ratio α (0.8 – 3.0), number of longitudinal stiffeners n (1 – 4), compression stress in the plate (0 – f_y).

300 calculations for different combinations of the parameters were performed and some of the results are given in a condensed form in Fig. 11 and Fig. 13.

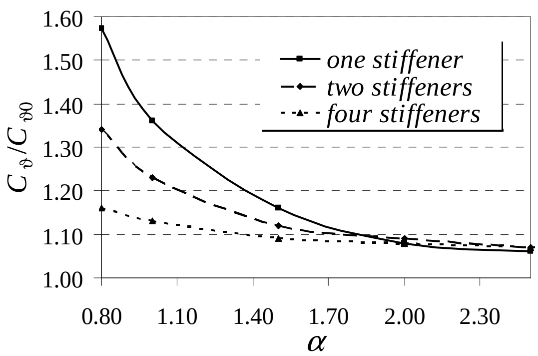


Fig. 11: Relation c_ϑ - α

In Fig. 11 the ratio $c_\vartheta/c_{\vartheta 0}$ is shown in relation to aspect ratio α for the plate without longitudinal compression stresses. The actual value of c_ϑ is larger than $c_{\vartheta 0}$ given by (44) and (45). The reason for such result is that in (44) and (45) real plate behaviour is neglected (see Fig. 5). By increasing α torsional stiffness gets smaller, approaching the value of $c_{\vartheta 0}$. Also the number of stiffeners plays an important role.

In Fig. 12 c_ϑ is normalised with the maximum value of c_ϑ from Fig. 11, and from the diagrams in Fig. 12 it is evident that for the selected stiffener topology the influence of the parameter α is not very important, except at small subpanel slenderness b/t. On the other hand, the slenderness parameter b/t plays an important role. The influence of slenderness is summarized in Fig. 13. Regarding the compression stresses in the plate, as expected in all cases, elastic torsional stiffness reduces with increasing stress.

Fig. 13 gives results for 1, 2 and 4 stiffeners at α = 2.0 for different slenderness parameters b/t. In the nondimensional form the influence of the number of the stiffeners is small and more pronounced only for small slenderness parameters b/t.

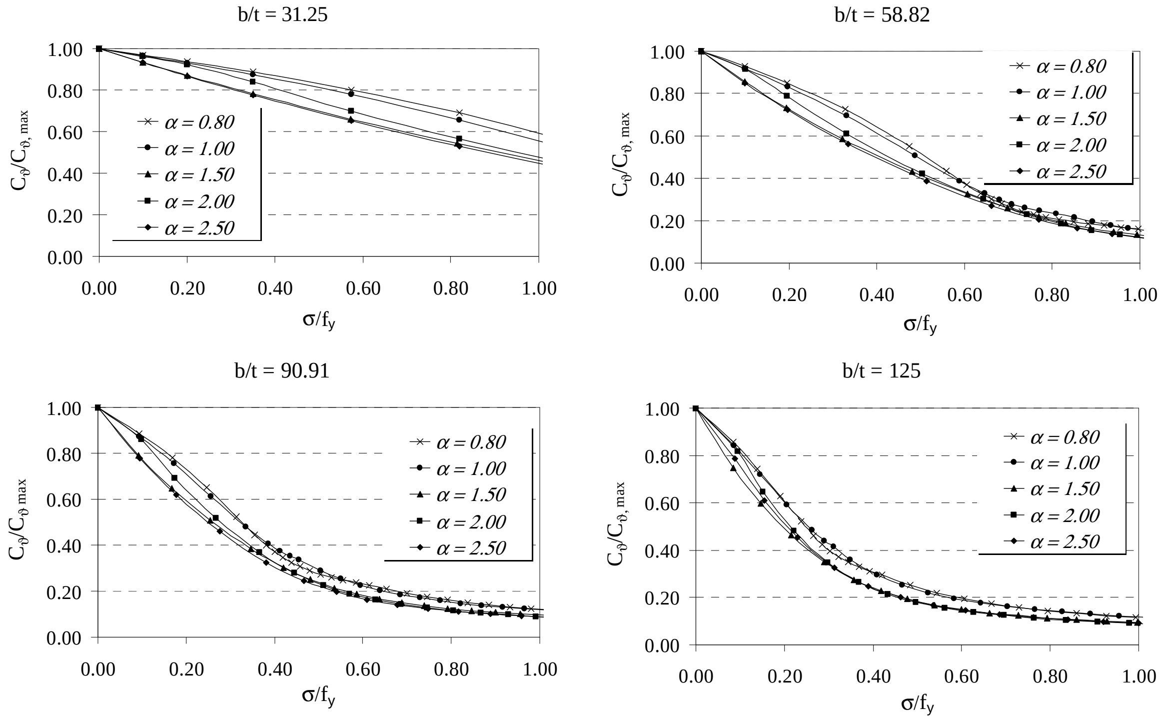


Fig. 12: Relation c_ϑ - compression stress at different subpanel slenderness and values of α for one stiffener

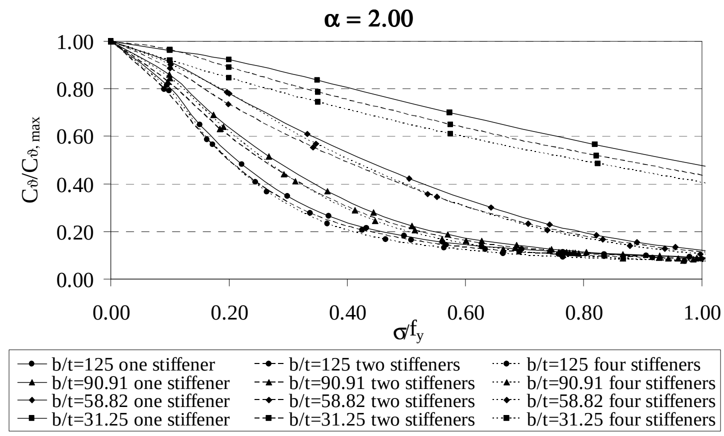


Fig. 13: Relation c_ϑ - compression force at α = 2 for 1, 2 and 4 stiffeners and different values of slenderness parameter b/t

4.4 ANALYTICAL EXPRESSIONS FOR C_ϑ

For a practical application simple analytical expressions were derived based on the results of numerical calculations.

It is obvious that all the main parameters should be included in the analytical expressions:

$$c_\vartheta = c_\vartheta\left(\alpha, \frac{b}{t}, \frac{\sigma}{f_y}\right) \qquad (46)$$

Numerical results lead us to the conclusion that two separate functions should be used, one to describe dependence on α and the number of stiffeners, and the other to cover parameters b/t and σ/f_y:

$$c_\vartheta = c_{\vartheta 0} \cdot f_1(\alpha) \cdot f_2\left(\frac{b}{t}, \frac{\sigma}{f_y}\right). \qquad (47)$$

f_1 differs with the number of stiffeners (see Fig. 11) and f_2 can be assumed to be independent of the number of the stiffeners.

Functions f_1 and f_2 were determined with the help of the computer code MATHEMATICA applying the least square method. For $f_1(\alpha)$ the following solution was obtained

$$f_1(\alpha) = \frac{1}{A \cdot \alpha^2} + \frac{1}{B \cdot \alpha} + C \qquad (48)$$

Coefficients A, B and C (rounded values) are listed in Table 1.

	Number of Stiffeners		
	1	2	4
A	3,0000	5,0000	50,0000
B	0,0000	0,0000	10,0000
C	1,0000	1,0000	1,0000

Table 1: Values of coefficients A, B and C from $f_1(\alpha)$

f_2 (b/t, σ/f_y) can be expressed as an exponential function. After some simplifications of the solution obtained by MATHEMATICA a possible solution may be

$$f_2\left(\frac{b}{t}, \frac{\sigma}{f_y}\right) = e^{-\frac{1}{30,9} \cdot \frac{b}{t} \cdot \frac{\sigma}{f_y}} \qquad (49)$$

Final expressions for elastic torsional restraint are given in Table 2.

In Fig. 14 and Fig. 15 a comparison between numerical and analytical values is shown and a good agreement is achieved for f_1 (Fig. 14) and f_2 (Fig. 15).

A correlation diagram between all the numerical results and the derived analytical expressions is plotted in Fig. 16. A good correlation with safe-sided results is obtained.

Further safe-sided simplification is possible by neglecting favourable plate behaviour shown in Fig. 11. f_1 is then taken as equal to 1 and for all panel topologies the elastic torsional stiffness can be written as

$$c_\vartheta = c_{\vartheta 0} \cdot e^{-\left(\frac{1}{30.9}\cdot\frac{b}{t}\cdot\frac{\sigma}{f_y}\right)} \tag{50}$$

which is a very simple expression suitable for everyday engineering practice.

Number of stiffeners	c_ϑ
1	$\frac{E\cdot t^3}{2\cdot b\cdot(1-\nu^2)}\left[\left(1+\frac{1}{3\cdot\alpha^2}\right)\cdot e^{-\left(\frac{1}{30.9}\cdot\frac{b}{t}\cdot\frac{\sigma}{f_y}\right)}\right]$
2	$\frac{E\cdot t^3}{2,5\cdot b\cdot(1-\nu^2)}\left[\left(1+\frac{1}{5\cdot\alpha^2}\right)\cdot e^{-\left(\frac{1}{30.9}\cdot\frac{b}{t}\cdot\frac{\sigma}{f_y}\right)}\right]$
3 or more	$\frac{E\cdot t^3}{3\cdot b\cdot(1-\nu^2)}\left[\left(1+\frac{1}{10\cdot\left(\alpha\cdot\frac{h}{5\cdot b}\right)}+\frac{1}{50\cdot\left(\alpha^2\cdot\frac{h^2}{25\cdot b^2}\right)}\right)\cdot e^{-\left(\frac{1}{30.9}\cdot\frac{b}{t}\cdot\frac{\sigma}{f_y}\right)}\right]$

Table 2: Expressions for c_ϑ for different number of stiffeners

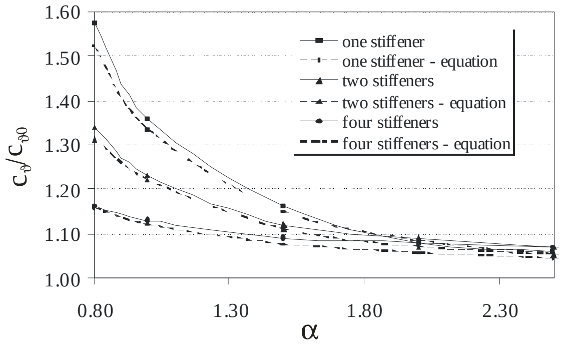


Fig. 14: Function f_1 in comparison to numerical results

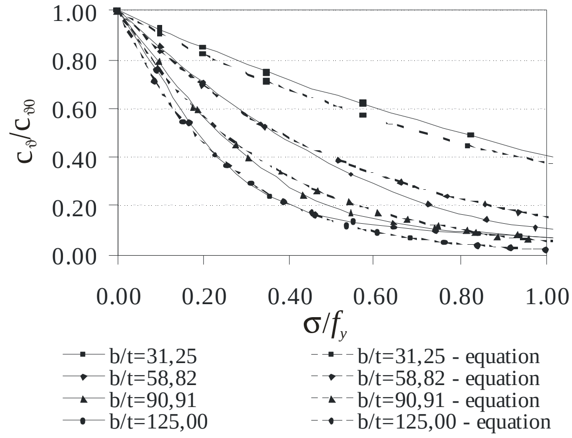


Fig. 15: Function f_2 in comparison to numerical results

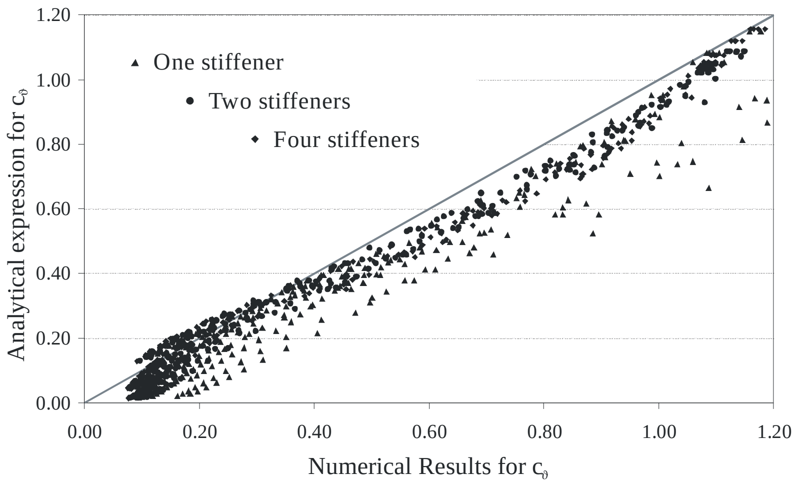


Fig. 16: Correlation between numerical and analytical results

5. CONCLUSIONS

The paper presents the design of rigid transverse stiffeners of plate girders according to Eurocode standard EN 1993-1-5. The background of the design rules from EN 1993-1-5 is presented and additional rules for single sided stiffeners are given. Although the design rules are based on strength and deformability checks taking account of second order effect and initial geometric imperfections, they can be presented in relatively simple formats, suitable for everyday design practice.

The paper also gives information on how to prevent torsional buckling of longitudinal and transverse stiffeners. For this purpose based on extensive numerical study, simple analytical expressions are derived for elastic torsional stiffness of the plate that may be used in the calculation of the critical torsional buckling stress of a stiffener. By doing this a critical buckling stress may be increased significantly and the design criterion (33) may be fulfilled more easily.

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