FINITE ELEMENT ELASTIC ANALYSIS OF HYPAR SHELLS ON WINKLER FOUNDATION

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1. ABSTRACT

In this paper, the hyperbolic paraboloidal shell is investigated. The two components of the interacting system; the soil and the shell foundation, are modelled using the finite element method. In this study, 9-node isoparametric degenerated shell element with five degrees of freedom at each node was used. The soil-structure interaction between the shell elements and the supporting medium are model in this study by representing the soil medium by certain analytical equivalent such as Winkler model with both normal compressional and tangential frictional resistances.

Parametric studies have been carried out to investigate the effect of some important parameters on the behaviour of shell foundations. These parameters are: shell thickness, shell warp, ridge and edge beams cross-sectional dimensions.

Comparison between the results obtained by the present analysis and those obtained by many other investigations are made. The present analysis shows satisfactory results when compared with those obtained by other studies with largest percentage difference of 4.4 % in the value of the vertical displacement.

2. INTRODUCTION

Shells are structures that derive their strength from *form* rather than *mass*. This form enables shells to put a minimum of material to maximum structural advantages. While a plain element like a roof slab undergoes bending when subjected to a vertical loads including self-weight, a shell which is non-planar or a spatial system sustains the applied loads primarily by direct in-plane or membrane forces (compression or tension). Bending forces, even when present, it normally assumes only a place of secondary importance. Among the shells, which have come into wider use in foundations, the hyperbolic paraboloid (or briefly hypar) shell has been the most important type. Besides its geometric simplicity, resulting from its straight-lines property, the hypar shell has high structural efficiency. Four such shell quadrants jointed together by a system of edge and ridge beams, the latter terminating at the column base [Fig. 1] have been widely used as column foundation in many parts of the world [1].

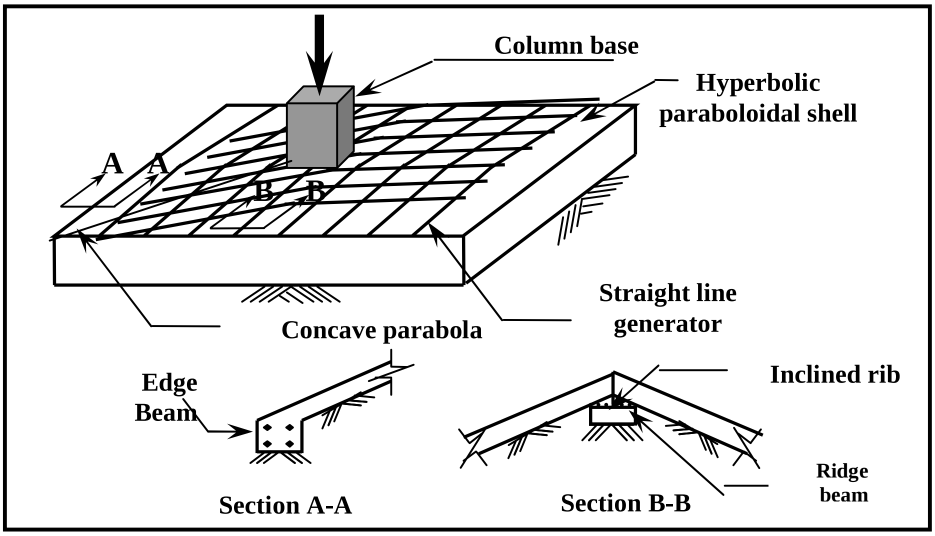

Fig. 1 : Individual hypar footing.

3. FINITE ELEMENT METHOD

The Ahmad-type *degenerated isoparametric* shell elements based on independent rotational and translational displacement interpolation have become popular in recent years. In this element, the Mindlin-type theory is employed. The normal to the middle surface of the three-dimensional element is constrained to remain straight after deformation in order to overcome the numerical difficulty associated with the large stiffness ratio in the through-thickness direction. Also, this element neglects the strain energy associated with stresses perpendicular to the local $x' - y'$ surface and constrains the normal stress component to zero to simplify the constitutive equations. By adopting the isoparametric geometric description, the element can be used to represent thin and thick shells with arbitrary shapes, circumventing the complexities of classical shell theory and differential geometry [2].

The four coordinate systems used in the degenerated shell element formulations are shown in Fig. 2.

(1) Global Cartesian coordinate system (x, y, z or x_i).

(2) Natural coordinate system (ξ,η,ζ).

(3) Local Cartesian coordinate system (x', y', z' or x_i').

(4) Nodal Cartesian coordinate system (V_1^k, V_2^k, V_3^k).

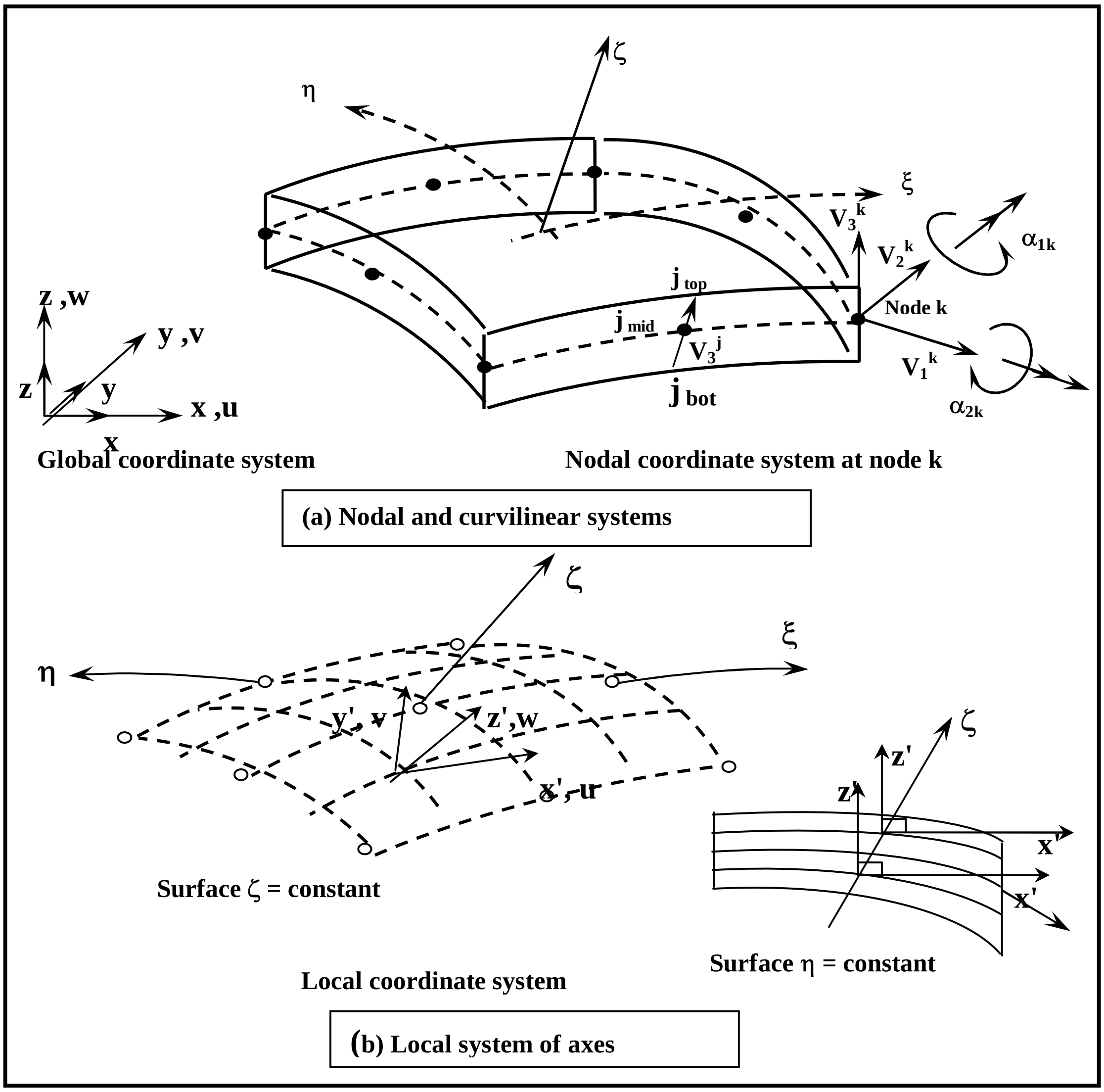

Fig. 2: Coordinate systems for the degenerated shell element.

4. ELASTIC FOUNDATION

For a foundation represented by Winkler model for both compressional and frictional resistances, the stiffness matrix is given by Al-Azzawi in 1995 [3]:

$$[K]_f = \begin{bmatrix} [R_w] & 0 & .. & 0 \\ 0 & [R_w] & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & [R_w] \end{bmatrix}_{n\times n} \qquad (1)$$

5. APPLICATIONS AND DISCUSSIONS

In this paper, an example have been analyzed by a computer program named **SFAP (S**hell **F**oundation **A**nalysis **P**rogram), which is developed from a program named **PLAST** [4] by adding subroutines of foundation properties and stiffnesses in order to be capable of solving different types of shell foundations.

5.1 Square Hyperbolic Paraboloidal Shell Foundation

Melerski in 1986 analyzed a square hypar shell foundation by the finite difference method [5]. The square hypar footing has 5m × 5m in plan with 0.5m rise and 0.1m thickness. The edge and ridge beams cross sectional dimensions are 0.2m × 0.3m and the footing is subjected to a central vertical load of 1600 kN. The elastic properties of the hypar shell are ($E = 24 \times 10^6$ kN/m^2 and $\nu = 0$). The footing is resting on a Winkler foundation with $K_x = K_y = K_z = 12000$ kN/m^3). The problem is solved by the finite element method and due to symmetry one quadrant of the shell is taken for the analysis. Due to symmetry, one quadrant of footing is analyzed by using 9-node isoparametric degenerated shell elements as shown in Fig. 3.

This problem was also solved by Hassan in 2002 who used a different finite element model [6]. In the present study, the results are compared with both Melerski and Hassan.

Figure 4 shows the variation of the vertical displacement along the diagonal of the footings. Figures 5 and 6 show the variation of axial force and the bending moment in the edge beam, respectively. Figures 7 and 8 show the variation of axial force and bending moment in ridge beam, respectively.

The percentage difference in results (for the vertical displacement) between the present study and Melerski is about 4.409 %. The percentage difference in results between the present study and Hassan is about 7.083 %.

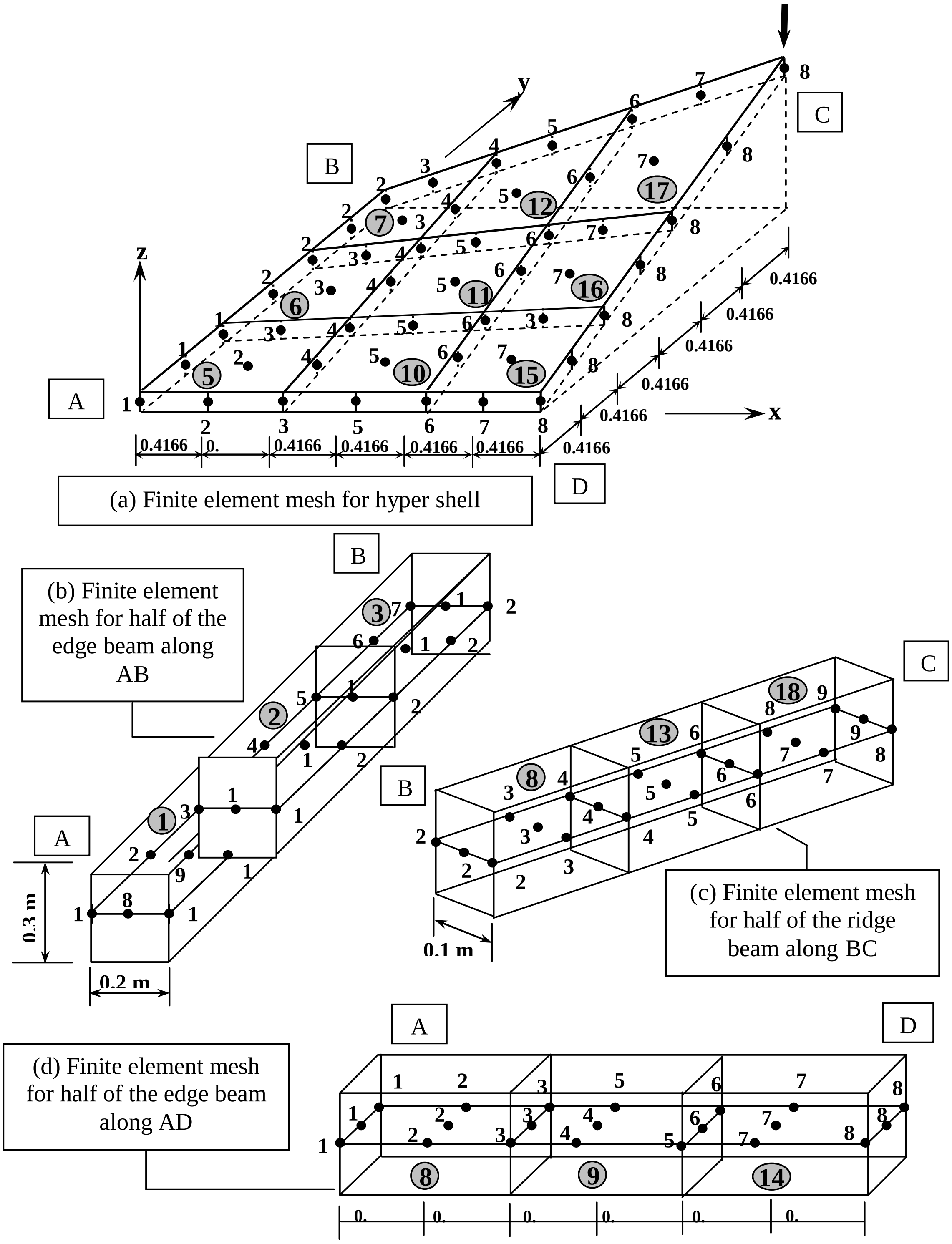

Fig. 3: Finite element mesh for hyperbolic paraboloidal footing with edge and ridge beams.

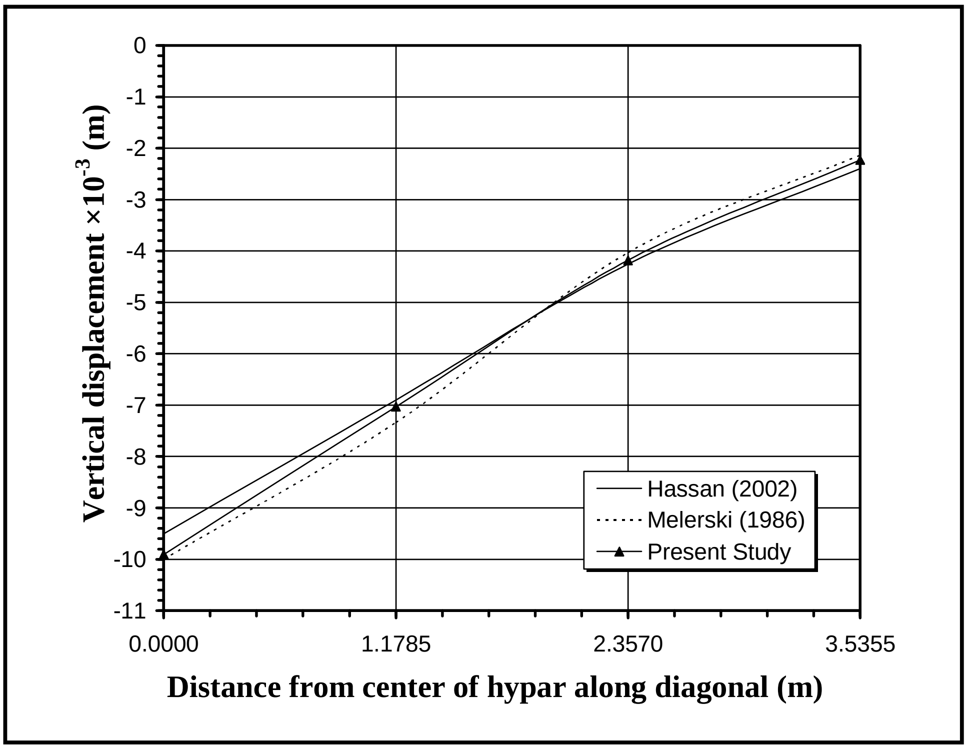

Fig. 4: Variation of vertical displacement along the diagonal of hypar shell.

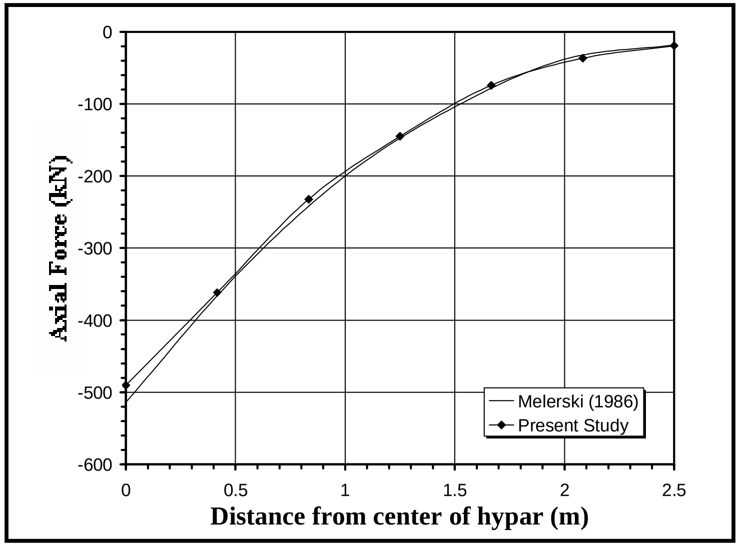

Fig. 5: Variation of axial force in ridge beam.

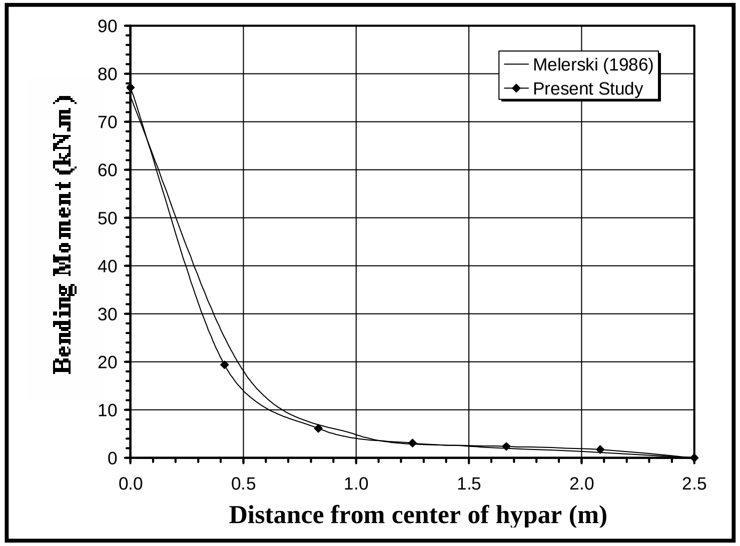

Fig. 6: Variation of bending moment in ridge beam.

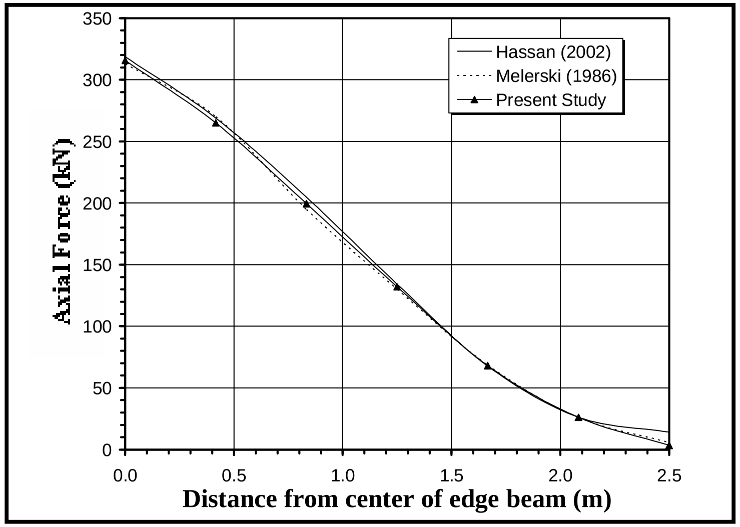

Fig. 7: Variation of axial force in edge beam.

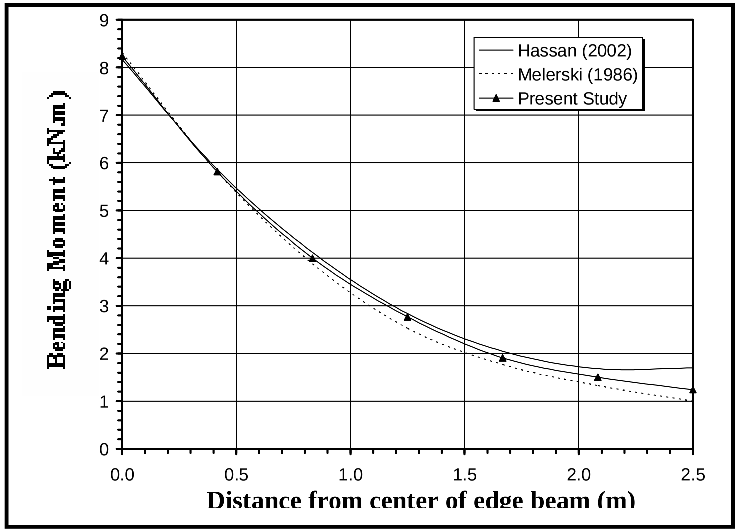

Fig. 8: Variation of bending moment in edge beam.

5.2 Parametric study

In order to study the influence of variation of selected parameters on the behavior of the hypar shell foundations, four parameters are considered which are: shell warp, shell thickness, ridge beam cross sectional dimensions and edge beam cross sectional dimensions.

a) Shell warp

The influence of variation of shell warp ($k = f/ab$) on its behavior is now considered. Different values are taken (k = 0.08, 0.16, 0.24, 0.32, 0.4 m^{-1}). Figure 9 shows the variation of the vertical displacement along the diagonal. From this figure, it is seen that with increasing the shell warp, the vertical displacement decreases near and at the center of the hypar shell while it increases when it approaches the edges. This behavior is due to the increase in the concentrated load component towards the edges of the shell with the increase in the shell warp.

Figures 10 and 11 show the variation of the bending moment and shear force in the ridge beam, respectively. From these figures, it can be noticed that the bending moment and the shearing force decreases with the increasing in the shell warp, this is due to the decreases in the concentrated load normal component which causes the bending moment and the shear force in the ridge beam. Figure 12 shows that the variation of the axial force in the ridge beam decreases. From this figure, it is seen that the axial force decreases with the increase in the shell warp which is due to reason that the increase in the shell warp will cause a decrease in the membrane shearing stress in the shell.

b) Shell thickness

The influence of variation of shell thickness is now considered. Five values of shell thickness were considered in this study (h = 0.1, 0.15, 0.2, 0.25 and 0.3m). Figure 13 shows the variation of the vertical displacement along the diagonal of the hypar footing. From this figure it is found that the vertical displacement decreases at the center of the footing (i.e. under the concentrated load) and increases near the edges. This behavior is the result of increasing the shell rigidity, i.e. the hypar shell tries to reduce the vertical displacement.

Figures 14 and 15 show the variation in the bending moment and the shearing force in the ridge beams, respectively. It is seen that the bending moments and the shearing forces decreases with the increase in the shell thickness which is due to the decrease in the variation of the vertical displacement. Figure 16 shows the variation of the axial force in the ridge beam. From this figure it is seen that the axial force decreases with the increase in the shell thickness.

c) Ridge beam cross sectional dimensions

Four values of ridge beam cross sectional dimensions are taken (0.20 m × 0.20 m, 0.20 m × 0.25 m, 0.20 m × 0.30 m and 0.20 m × 0.35 m). Figure 17 shows the variation of the vertical displacement along the ridge beam. It is seen that with increasing the cross sectional dimensions of the ridge beam, the vertical displacement at the center of the hypar shell will decrease. This behavior is due to the four ridge beams meeting at the center of the hypar shell and they will increase the hypar shell stiffness and so to decrease the vertical displacement at the same position.

Figures 18, 19 and 20 show the variation of the bending moment, shear force and axial force in the ridge beam, respectively. The values of the bending moment, shearing force and axial force in the ridge beam will increases with increasing its cross sectional dimensions. This behavior is due to the fact that the ridge beam stiffness is increased and will attract more forces.

d) Edge beam cross sectional dimensions

Four values of edge beam cross sectional dimensions are taken (0.20 m × 0.20 m, 0.20 m × 0.25 m, 0.20 m × 0.30 m and 0.20 m × 0.35 m). Figure 21 shows the variation of the vertical displacement along the edge beam. It is seen that with increasing the cross sectional dimensions of the edge beam, the vertical displacement at the center of the beams will decrease while it will increase near edges. This behavior is due to the increase in the edge beam rigidity by increasing its cross sectional dimensions.

Figures 22 and 23 show the variation of the bending moment and the shear force in the edge beam. It is seen that with increasing the cross sectional dimensions of the edge beam, the bending moment and the axial force in the edge beam will increase which is due to the increase in the beam stiffness. Figure 24 shows the variation of axial force in the edge beam. It is seen that the shear force increases at the center of the beam with increasing its cross sectional dimensions which is due to the increase in its stiffness.

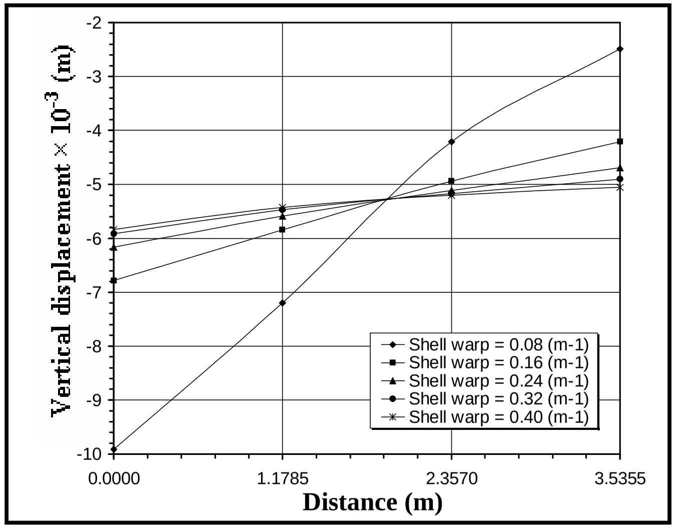

Fig. 9: Effect of variation of shell warp on the vertical displacement along the diagonal.

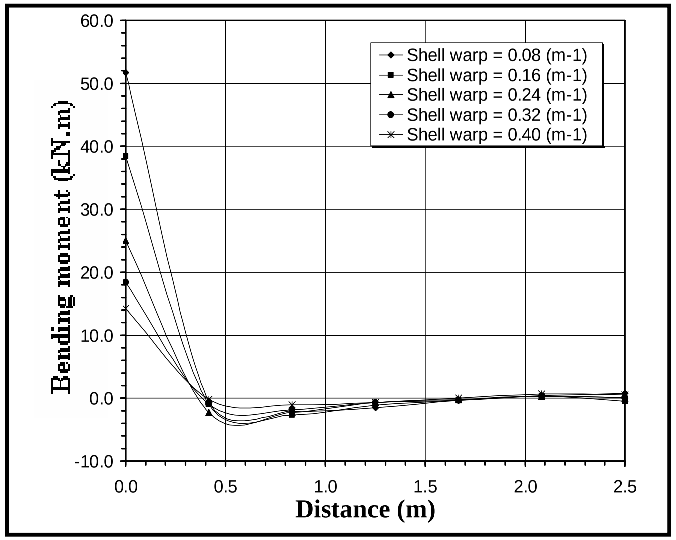

Fig. 10: Effect of variation of shell warp on the bending moment in ridge beam.

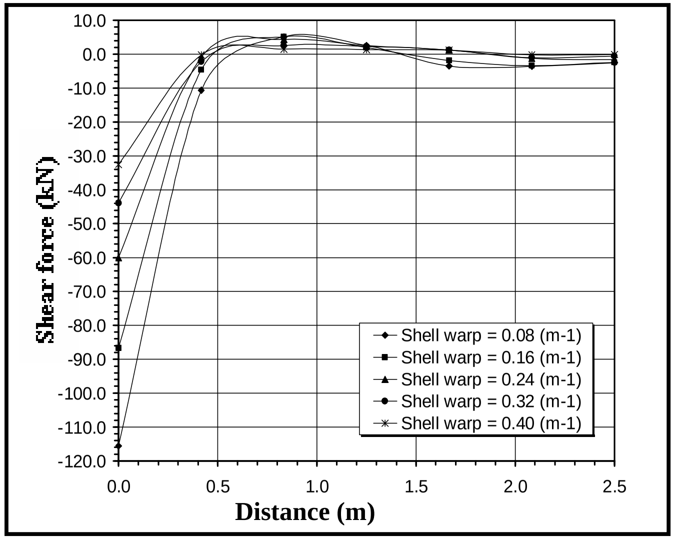

Fig. 11: Effect of variation of shell warp on the shear force in ridge beam.

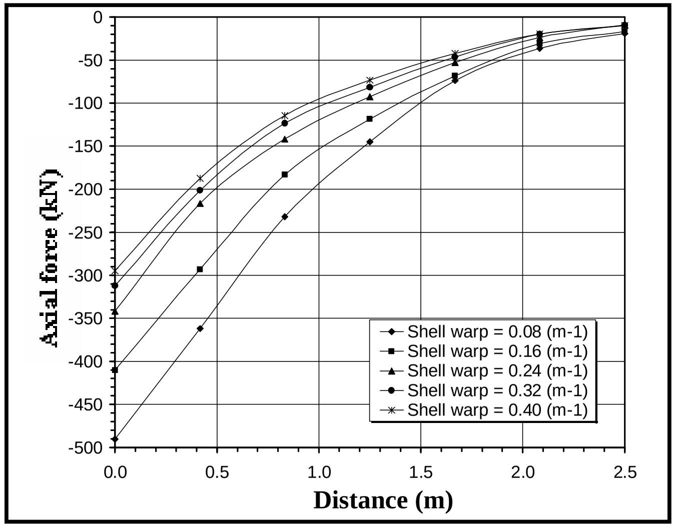

Fig. 12 :Effect of variation of shell warp on the axial force in ridge beam.

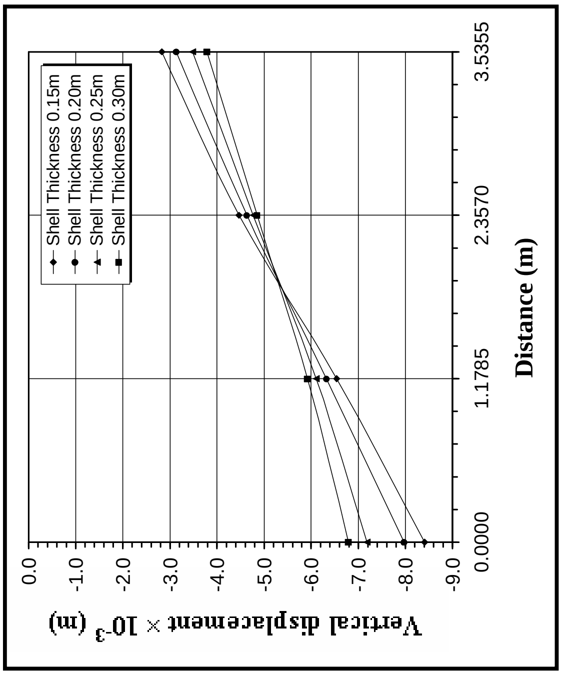

Fig. 13: Effect of variation of shell thickness on the vertical displacement along the diagonal.

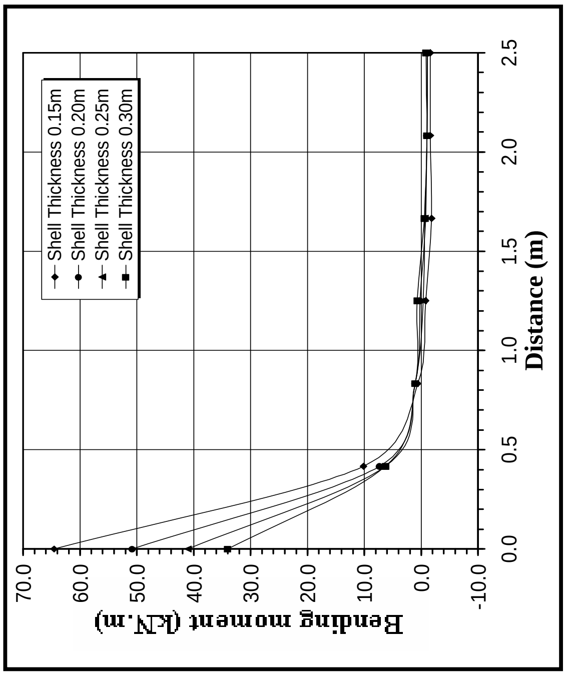

Fig. 14: Effect of variation of shell thickness on the bending moment in ridge beam.

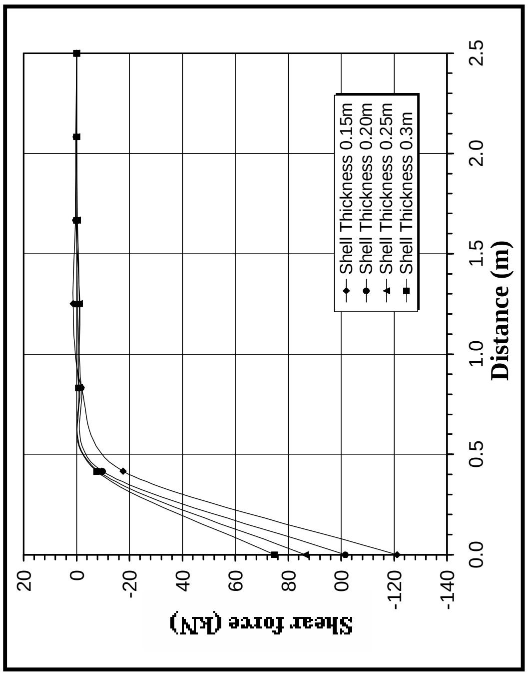

Fig. 15: Effect of variation of shell thickness on the shear force in ridge beam.

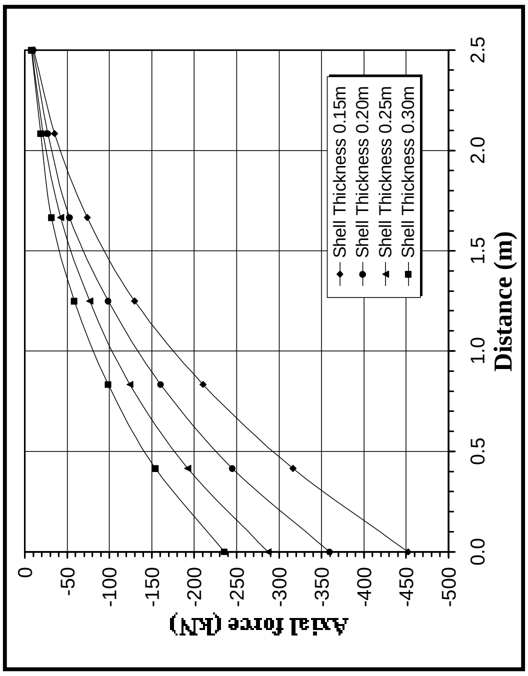

Fig. 16: Effect of variation of shell thickness on the axial force in ridge beam.

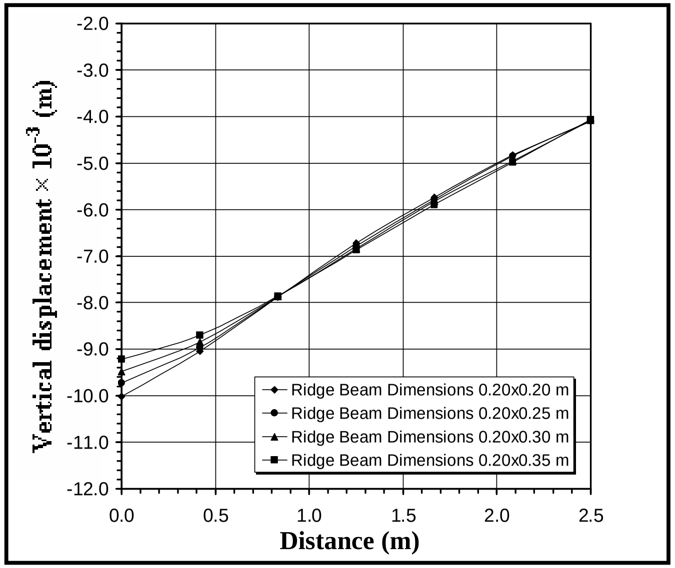

Fig. 17: Effect of variation of ridge beam cross sectional dimensions on the vertical displacement along ridge beam.

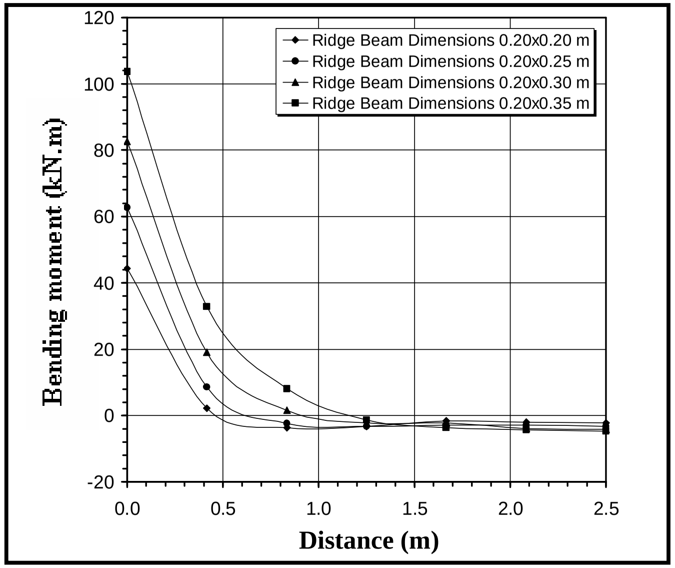

Fig. 18: Effect of variation of ridge beam cross sectional dimensions on the bending moment in ridge beam.

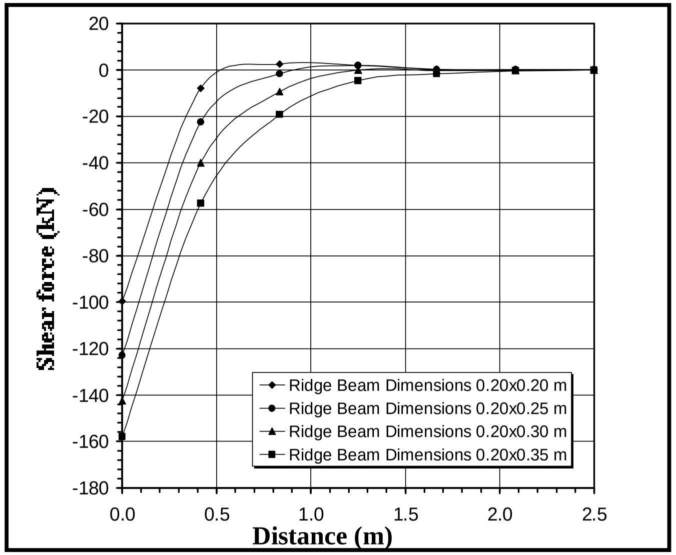

Fig. 19: Effect of variation of ridge beam cross sectional dimensions on the shear force in ridge beam.

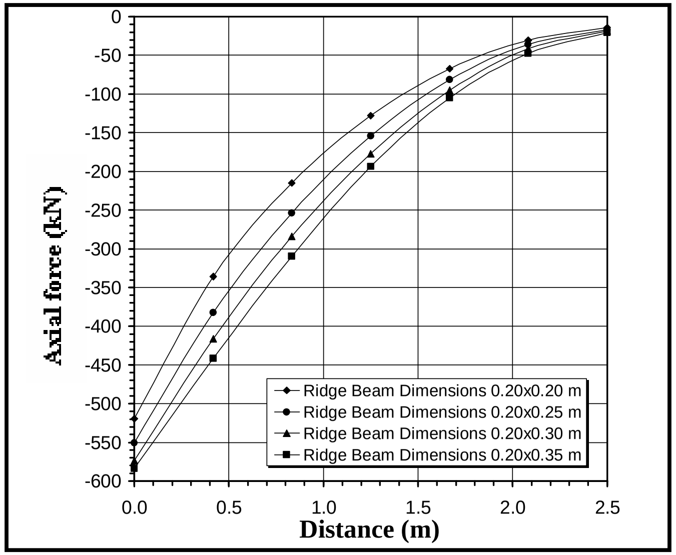

Fig. 20: Effect of variation of ridge beam cross sectional dimensions on the axial force in ridge beam.

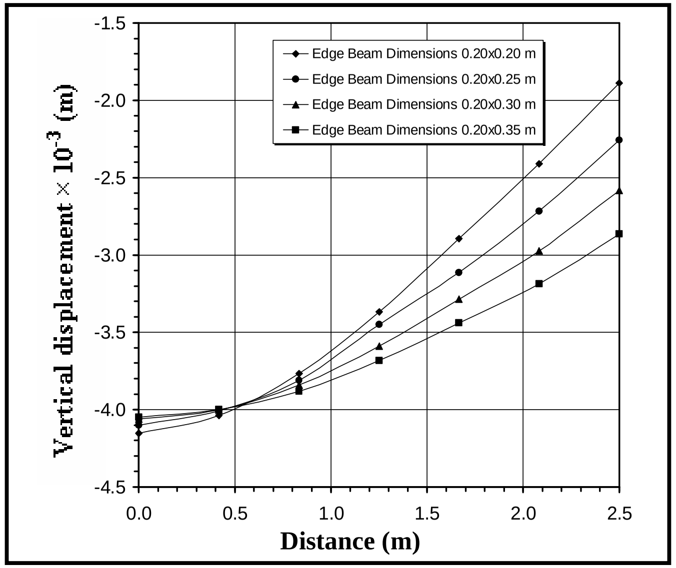

Fig. 21: Effect of variation of edge beam cross sectional dimensions on the vertical displacement along edge beam.

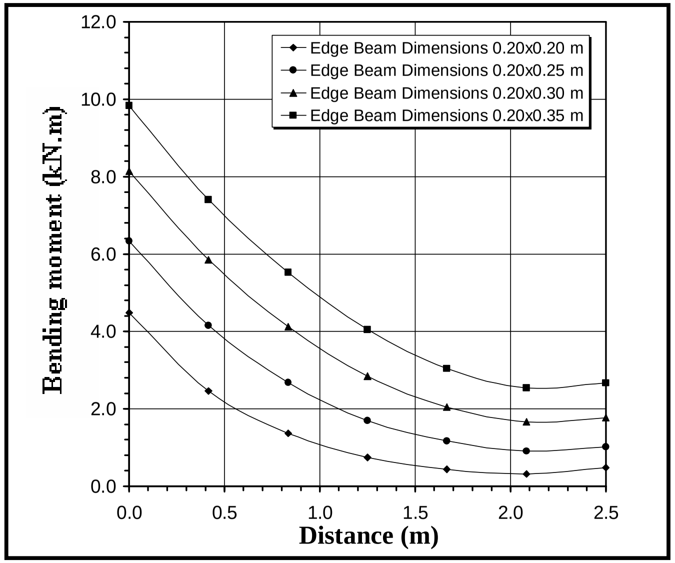

Fig. 22: Effect of variation of edge beam cross sectional dimensions on the bending moment in edge beam.

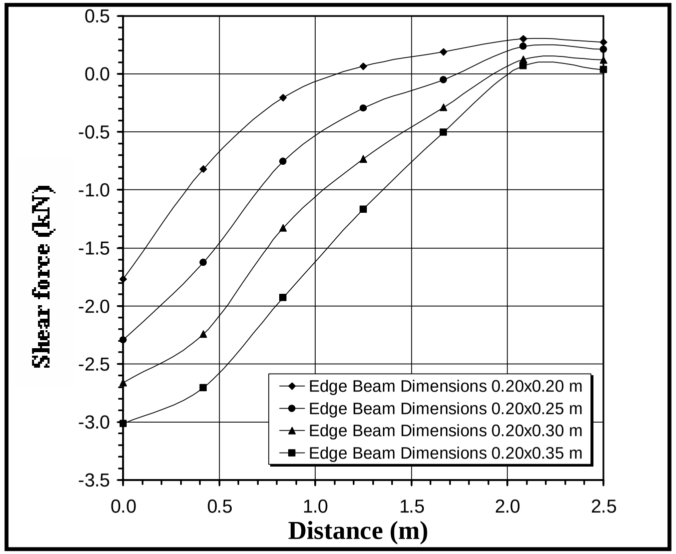

Fig. 23: Effect of variation of edge beam cross sectional dimensions on the shear force in edge beam.

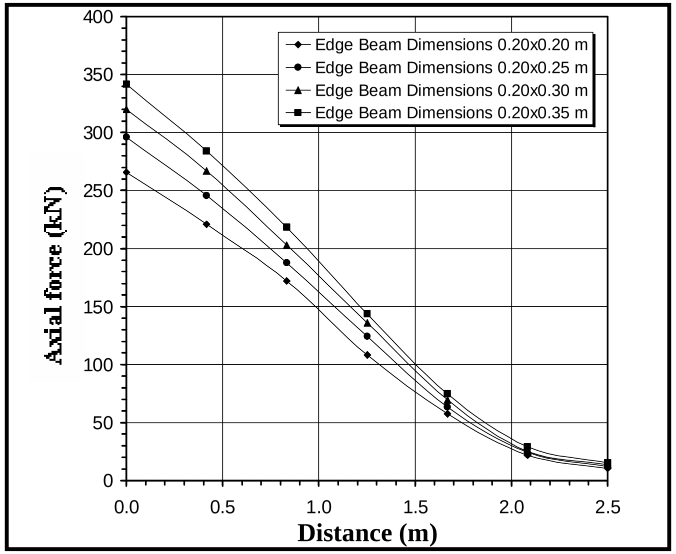

Fig. 24: Effect of variation of edge beam cross sectional dimensions on the axial force in edge beam.

6. CONCLUSIONS

The major conclusions obtained from the study of the hypar shell foundation are:

1. Increasing the shell warp from 0.08 m^{-1} to 0.16 m^{-1}.

a) decreases the vertical displacement by about (31.5 %) near and at the center of the hypar shell while it increases by about (70.11 %) when it approaches the edges.
b) decreases the bending moments in the ridge beams by about (53.53 %).
c) decreases the shearing forces in the ridge beams by about (24.94 %).
d) decreases the axial forces in the ridge beams by about (16.33 %).

2. Increasing shell thickness from 0.1 m to 0.15 m.

a) decreases the vertical displacement by about (5.2 %) at the center of the hypar shell while it increases by about (10.78 %) when it approaches the edges.
b) decreases the bending moments in the ridge beams by about (21.22 %).
c) decreases the shearing forces in the ridge beams by about (15.97 %).
d) decreases the axial force in the ridge beams by about (20.48 %).

3. Increasing edge beam cross sectional dimensions from 0.2 m × 0.2 m to 0.2 m × 0.25 m.

a) decreases the vertical displacement by about (1.25 %) at the center of the edge beams while it increases about (19.53 %) when it approaches the edges.
b) increasing the bending moments in the edge beams by about (41.46 %).
c) increasing the axial forces in the edge beams by about (11.43 %).
d) increasing the shearing forces by about (29.59 %) at the center of edge beams while it decreases by about (23.60 %) at the ends of those beams.

4. Increasing ridge beam cross sectional dimensions from 0.2 m × 0.2 m to 0.2 m × 0.25 m.

a) decreases the vertical displacement by about (2.95 %) at the center of the hypar shell foundation.
b) increasing the bending moments in the ridge beams by about (41.42 %).
c) increasing the shearing forces in the ridge beams by about (23.53 %).
d) increasing the axial forces in the ridge beams by about (6.06 %).

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ΕΛΑΣΤΙΚΗ ΑΝΑΛΥΣΗ ΜΕ ΤΕ ΜΕΘΟΔΟ ΤΩΝ ΠΕΠΕΡΑΣΜΕΝΩΝ ΣΤΟΙΧΕΙΩΝ ΚΕΛΗΦΩΤΗΣ ΘΕΜΕΛΙΩΣΗΣ ΜΕ ΜΟΡΦΗ ΝΠΕΡΒΟΛΙΚΟΝ ΠΑΡΑΒΟΛΟΕΙΔΟΝς ΕΔΡΑΖΟΜΕΝΟ ΣΕ ΕΛΑΤΗΡΙΩΤΟ ΕΔΑΦΟΣ WINKLER

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ΠΕΡΙΛΗΨΗ

Στην παρούσα εργασία μελετάται κελυφωτή θεμελίωση με μορφή υπερβολικού παραβολοειδούς. Οι δύο συνιστώσες του συστήματος που αλληλοεπιδρά, δηλαδή το έδαφος και η κελυφωτή θεμελίωση, προσομοιώνονται χρησιμοποιώντας τη μέθοδο των Πεπερασμένων Στοιχείων. Γίνεται χρήση ενός εκφυλισμένου ισοπαραμετρικού στοιχείου κελύφους εννέα κόμβων με πέντε βαθμούς ελευθερίας ανά κόμβο. Η αλληλεπίδραση εδάφους - κατασκευής μεταξύ των στοιχείων κελύφους και του υποκείμενου εδάφους προσομοιώνεται αντιπροσωπεύοντας το έδαφος με το αναλυτικό ισοδύναμο προσομοιώμα του ελατηριωτού εδάφους Winkler λαμβάνοντας υπόψη κάθετες θλιπτικές αντιστάσεις και εφαπτομενικές αντιστάσεις τριβής.

Πραγματοποιούνται παραμετρικές αναλύσεις με σκοπό να αποτιμηθεί η επίδραση διαφόρων σημαντικών παραμέτρων στη συμπεριφορά της κελυφωτής θεμελίωσης. Οι παράμετροι αυτοί είναι: το πάχος του κελύφους, η κλίση του κελύφους και οι διαστάσεις των διατομών των ακραίων και μεσαίων δοκών.

Τα αποτελέσματα των αναλύσεων συγκρίνονται με μια σειρά αποτελεσμάτων αντίστοιχων εργασιών και διαπιστώνεται ικανοποιητική σύγκλιση. Η μέγιστη ποσοστιαία απόκλιση προκύπτει για την τιμή της κατακόρυφης μετακίνησης και είναι της τάξεως του 4.4 %.