A COMPUTATIONAL METHOD BASED ON A STIFFNESS APPROACH FOR METAL STRUCTURES CONTAINING CABLE-ELEMENTS

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1. SUMMARY

This paper presents a simple numerical procedure for the static analysis of linearly elastic metal structures, containing cable-like members. The procedure is based on a version of the direct Stiffness Method of Structural Analysis and on a relative Equivalence Principle, proposed by G. Nitsiotas and holding for problems with unilateral constraints. The Finite Element Method is used and a linear complementarity problem with a reduced number of unknowns is finally solved.

2. INTRODUCTION

Structures containing cable-like members appear very often in the praxis of Civil Engineering, see e.g. recently in Greece the steel structures in the Athens Olympic Campus (Olympic Games 2004) or the stay-cable system of the Rion-Antirion bridge [1,2].
The problem of such structures containing cable-like members has its origin in the design of some metal structures, such as bridge trusses with counters or lattice struts with counter-diagonals. As well-known, the cable-like members can undertake tension but buckle and

become slack and structurally ineffective when subjected to a sufficiently large compressive force. Thus the governing conditions take an equality as well as an inequality form. So, the problem of structures containing as above cable-like members belongs to the so-called Inequality Problems of Mechanics, as their governing conditions are of both, equality and inequality type [3-6]. It is a high non-linear problem, requiring special treatment techniques [12-14].
In the early analysis attempts, many of these structures have been analyzed by a trial-and-error technique requiring repeated analysis of the structures for various loading systems. However, it should be noted that convergence to the correct solution by such iterative procedures is not always guaranteed. A more realistic treatment of the problem has been obtained by quadratic programming methods [3-4]. Further, the variational or hemivariational inequality concept has been used for the rigorous mathematical investigation of the problem [5,9]. The early numerical realizations of these approaches were based mainly upon the principle of minimum complementary energy [3,6]. Thus, an equivalence principle for the analysis of statically undetermined structures with unilateral constraints has been proven by G. Nitsiotas [3].
On the other hand, most of the available computer programs are based on the Displacement Method of structural Analysis. Consequently it seems more advantageous to combine the afore-mentioned equivalence principle with the displacement method to obtain a simple numerical procedure for the analysis of such structures.

The aim of this paper is to deal with the development of a simple numerical procedure for the static analysis of linearly elastic metal structures containing cable-like members by using a version of the direct Stiffness (Displacement) Method of Structural Analysis. The present procedure is based on the Finite Element Method and the Equivalence Principle, proposed by G. Nitsiotas in [3]. Using this principle, the analysis of such structures can be reduced to a Linear Complementarity Problem (LCP), which can be solved by various effective quadratic programming algorithms. A numerical example shows the direct applicability on the computer and the effectiveness of the procedure presented herein.

3. METHOD OF ANALYSIS

3.1 The Problem Formulation

A linearly elastic metal structure containing N cable-like members is considered. The structure is discretized according to the Finite Element Method. For the cables, pin-jointed bar elements with unilateral behavior are used. This unilateral behavior for the i-th cable-element (i = 1,…,N) is expressed by the following relations [3,10]:

$$e_i = F_{0i} \, . \, s_i + e_{i0} - v_i \tag{1}$$

$$s_i \geq 0 \,, \quad v_i \geq 0 \,, \quad s_i \, v_i = 0 \,. \tag{2a,b,c}$$

Here e_i, F_{0i}, s_i, e_{i0} and v_i denote the strain (elongation), natural flexibility constant, stress (tension), initial strain and slackness, respectively. From (1) it is clear that the slackness vi can be considered as an unknown initial strain which constitutes a reversible negative elongation [3]. Further, relations (2) express that either a non-negative stress-tension or a non- negative slackness exists on any cable.

For the remaining structure (besides the cables), the usual linearly elastic finite element models, which exhibit a bilateral behaviour, are used.

3.2 The Equivalence Principle and Stiffness Approach

Now the Equivalence Principle, proposed by G. Nitsiotas in [3], is applied for the whole structure. According to this principle, the structure under consideration behaves statically as an equivalent, linearly elastic structure, under the condition that in each cable-element either a non-negative stress or a fictitious, unknown, non- negative slackness appears-see rels (2). Thus, collecting in (Nx1) vectors **t** and **v** the stress and slackness behaviour of all the N cable-elements, corresponding, the following Linear Complementarity Conditions hold:

$$\mathbf{t} \geq \mathbf{0}\ , \quad \mathbf{v} \geq \mathbf{0}\ , \quad \mathbf{t}^{\mathbf{T}}\,\mathbf{v} = 0\ . \qquad (3a, b, c)$$

Further, following the Stiffness (Displacement) Method of Structural Analysis, we consider the cable-element as solidified rods and we assume that the so-modified structure is a statically stable one with bilateral rod-elemenets. So, the tension vector t is decomposed as follows [10]:

$$\mathbf{t} = \mathbf{C}\,.\,\mathbf{v} + \mathbf{t_0} \qquad (4)$$

Here $\mathbf{t_0}$ is the stress vector of the solidified cable-elements, now acting as normal bilateral rods, due to external actions and **C** is the influence matrix of **v** on **t** . For both it is assumed a linearly elastic, bilateral behavior for the stable structure, where. the cables are considered as already solidified bars. So, the 'natural' stiffness matrix **C** is symmetric and in general positive semi-definite.

Thus, if $\mathbf{t_0}$ and **C** are known, then vectors **t** and **v** can be determined by solving the *Linear Complementarity Problem* (LCP) formed by relations (3) and (4). For the solution of this problem, various effective algorithms are available [9]. Most of these algorithms reduce the above linear complementarity problem to a quadratic programming one [3-8] of the form:

$$\text{Min}\{\ (1/2)\ \mathbf{v}^{\mathbf{T}}\,\mathbf{C}\,.\,\mathbf{v} + \mathbf{v}^{\mathbf{T}}\,\mathbf{t_0}\ /\ \text{s.t.}\ \ \mathbf{v} \geq \mathbf{0}\ \} \qquad (5)$$

So, e.g., the sign constraints (3b,c) being the only side-conditions, this problem can be solved easily by the algorithm of Hildreth and D'Esopo [3, 8].

After the previous preparation we can now formulate the following numerical procedure for the static analysis of metal structures containing cable like members:
- a) Considering the cables as having been solidified (normal bilateralbars), the vector t0 due to external actions is determined by the Finite Element Method.
- b) Under the same assumption and by the same method as in (a), the influence matrix **C** is determined. In this matrix, Cij is the stress (axial force) in the solidified cable-element i caused by a unit-shortening vj = "1" imposed in the solidified cable-member j, (i,j = 1,…,N).

- c) The Linear Complementarity Problem of rels. (3) and (4) is solved to provide the sought vector **v**. So it is computed which cable-elements are activated (under tension) and which are not (under non-zero slackness).
- d) The final stress state of .the structure is determined by taking into account the external actions and the computed forces **t** of the active cable-elements.

Thus, the whole procedure requires the linear elastic analysis of the modified (with solidified cable-elements) structure (N+2) times, where N is the number of the cables, and the solution of a quadratic programming problem or a LCP. Alternatively, after having computed t, the structure is analyzed due to external actions by omitting the slack cables for which stage (c) has given zero tension values.

4. NUMERICAL EXAMPLE

The presented method is applied to a problem which is simple enough to permit reasonable assessment of the results, and realistic enough to demonstrate applicability and effectiveness of the method.
As shown in Fig. 1, the example problem considers a steel plane frame structure, with elastic modulus $E = 21.10^7$ KN/m^2 , reference bending stiffness EI_c = 6300 kNm^2 and ten (N = 10) cable members with cross-sectional area Fr = 5 cm^2 . These cables are placed as counter - diagonals and it is not known in advance which of them are activated or not by the given static loads or by a dynamic excitation, e.g. earthquake one. The horizontal loads correspond to the so-called “equivalent static loading” according to Greek Aseismic Code (2000) and to wind loads.

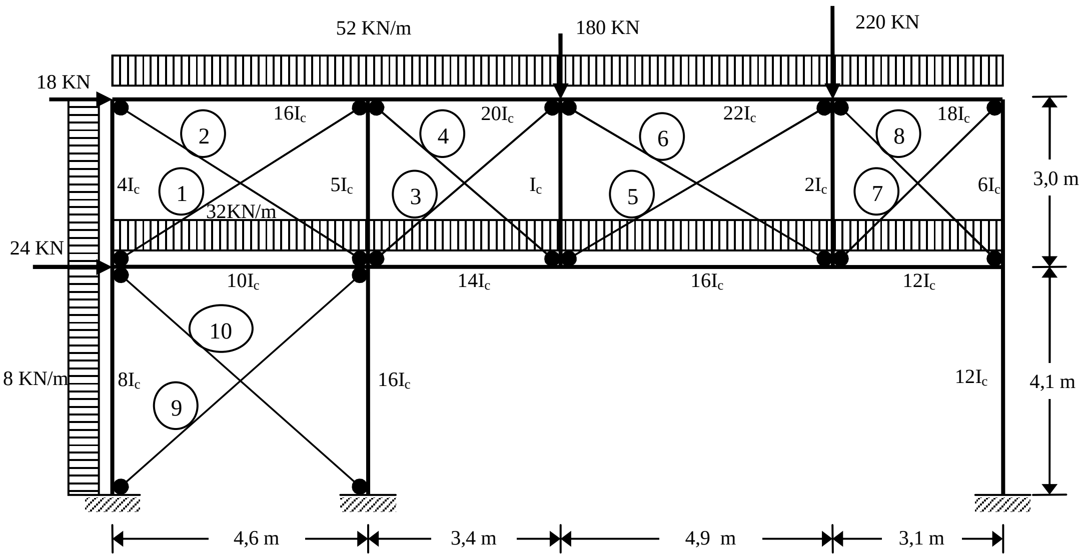

Fig. 1. The steel frame strengthened with 10 cable-elements.

The application of the presented numerical procedure gives first the values of the slackness of the no activated cable-elements:

$v_1 = 0{,}8877 \cdot 10^{-3}$ m, $v_3 = 11{,}7632 \cdot 10^{-3}$ m, $v_5 = 11{,}0132 \cdot 10^{-3}$ m, $v_8 = 11{,}0478 \cdot 10^{-3}$ m, $v_{10} = 1{,}2988 \cdot 10^{-3}$ m.

Further, the elements of the cable-stress vector **t**, where:

$$\mathbf{t} = [S_1, S_2, \ldots, S_{10}]^T,$$

are computed to have the following values (in kN):

$S_1 = S_3 = S_5 = S_8 = S_{10} = 0.0,$

and

$S_2 =$ 16,76 kN, $S_4 =$ 272,11 kN, $S_6 =$ 19,96 kN, $S_7 =$ 268,65 kN, $S_9 =$ 21,86 kN.

Thus, cables 2,4,6,7 and 9 are the only ones active, having zero slackness. These cables remain, whereas the other cables 1,3,5,8 and 10 can be considered as eliminated (or having $F_r = 0$).

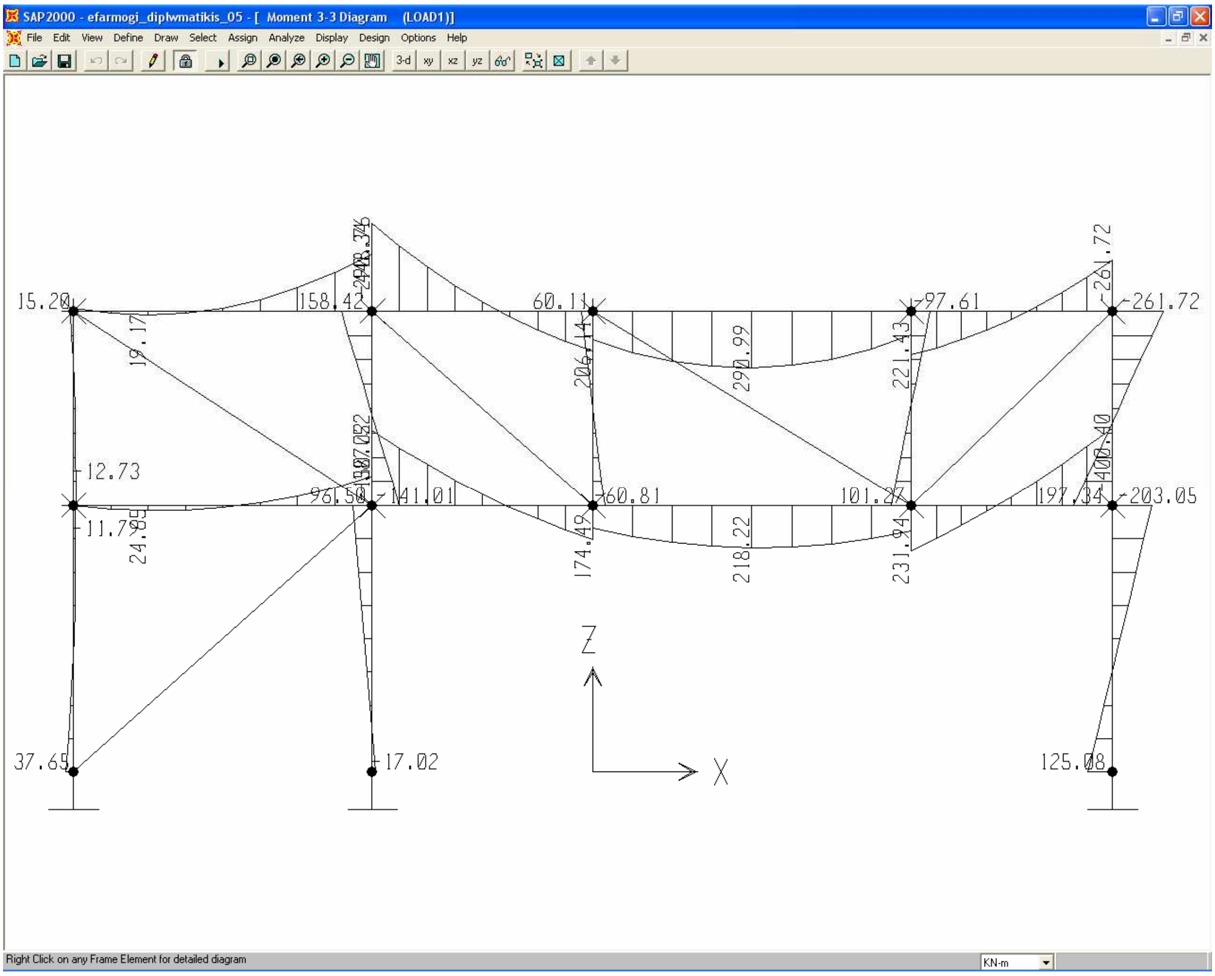

Fig. 2. SAP2000: Bending Moments Diagramme (in kNm) for the frame with the 5 active cable-elements.

Finally, by using SAP2000 [11], the final stress state is computed. In Fig. 2 is shown indicatively the final Bending Moments Diagram for the frame containing the active cable-elements only.

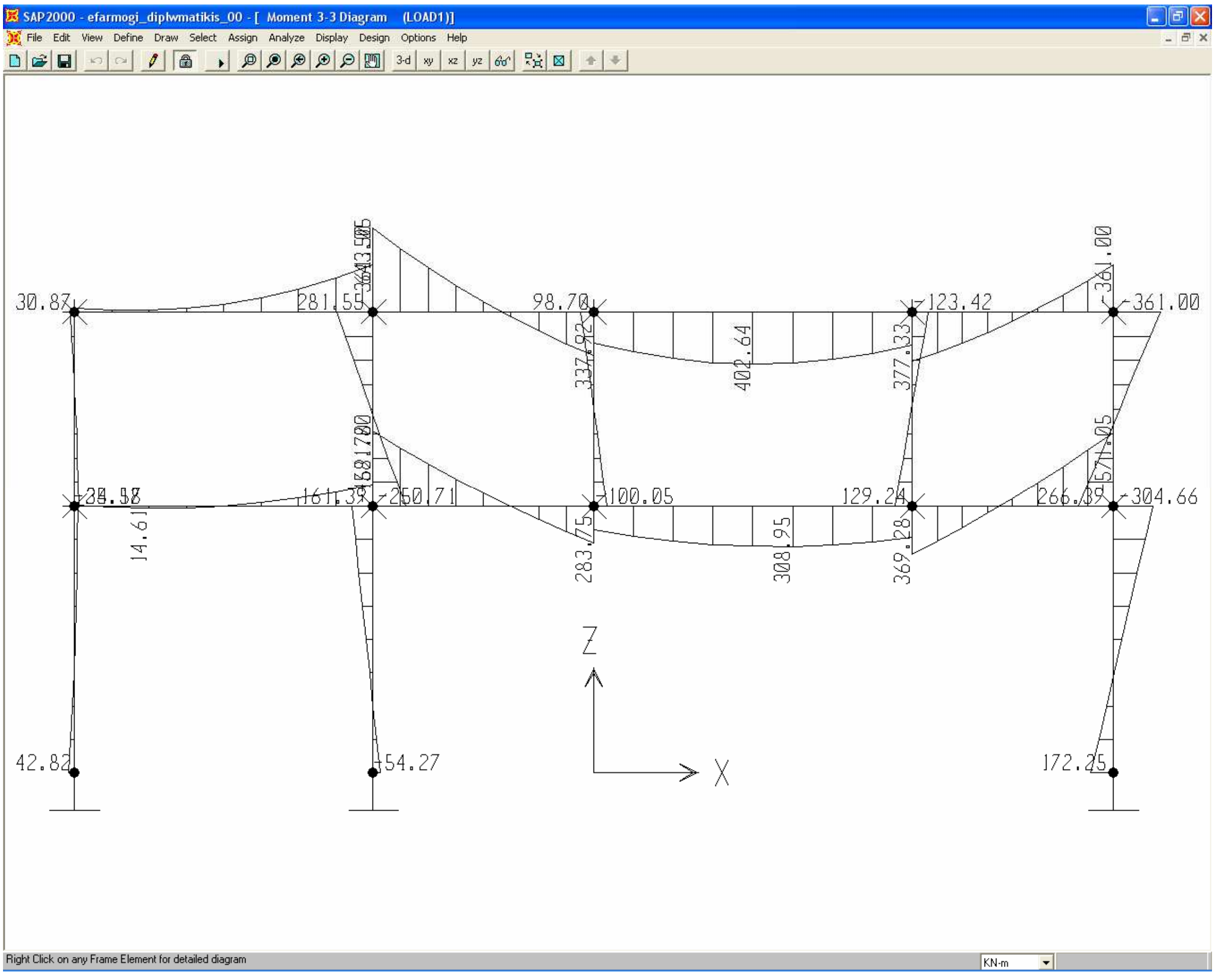

Fig. 3. SAP2000: Bending moments diagramme (in kNm) for the bare frame (without cable-elements).

For comparison reasons, in Fig. 3 is shown indicatively the final Bending Moments Diagram for the frame without cable-elements (bare frame).

5. CONCLUDING REMARKS

A simple numerical procedure for the static analysis of metal structures, containing cable-like members, has been presented herein. The procedure is based on the direct application of an Equivalence Principle proposed by G. Nitsiotas in [3] for such structures and on a version of the Direct Stiffness Method of Structural Analysis.
As it has been proved in an example problem, the numerical implementation of the procedure can be easily obtained by using generally available programs of the finite element method and of optimization (quadratic programming). So it can be computed which of the cable-elements are activated and which are not in response to the acting loading system.

6. REFERENCES

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ΜΙΑ ΑΡΙΘΜΗΤΙΚΗ ΕΠΙΛΥΣΗ ΜΕ ΤΗΝ ΜΕΘΟΔΟ ΔΥΣΚΑΜΨΙΑΣ ΜΕΤΑΛΛΙΚΩΝ ΦΟΡΕΩΝ ΜΕ ΚΑΛΩΔΙΑ

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ΠΕΡΙΛΗΨΗ

Παρουσιάζεται μια πρακτικά απλή και εύχρηστη αριθμητική επίλυση με την Άμεση Μέθοδο Δυσκαμψίας μεταλλικών φορέων που περιέχουν καλωδιωτά δομικά στοιχεία. Πρόκειται για φορείς με μονόπλευρους συνδέσμους που εμφανίζουν μεταβλητό τρόπο λειτουργίας. Αρχικά γίνεται η διατύπωση του προβλήματος σαν ανισοτικό πρόβλημα της Μηχανικής των Κατασκευών. Οι συνθήκες που διέπουν το πρόβλημα είναι τόσο ισότητες όσο και ανισότητες. Επισημαίνεται ότι πρόκειται για προβλήματα φορέων που είναι μη γραμμικά, με ιδιαίτερες δυσκολίες τόσο στην μαθηματική τους προσομοίωση όσο και στην αριθμητική τους επίλυση. Ακολούθως, για την αριθμητική επίλυση του προβλήματος, χρησιμοποιείται η Μέθοδος Άμεσης Δυσκαμψίας (Direct Stiffness Method) των Πεπερασμένων Στοιχείων (Finite Element Method – F.E.M.) σε συνδυασμό με μια μέθοδο βελτιστοποίησης. Τέλος, η όλη μεθοδολογία εφαρμόζεται σε μια χαρακτηριστική πρακτική περίπτωση ενός μεταλλικού φορέα ενισχυμένου με διαγώνιους αντισεισμικούς συνδέσμους από καλώδια.