Creep in aluminium structures

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1. ABSTRACT

Deflections of simple aluminium structures due to creep when the structure is subjected to fire, could be important. Their explicit calculation is necessary under certain conditions of time and temperature exposure (Eurocode EN 1999-1-2, [1]). Their calculation is, in general, cumbersome and demands the application of the appropriate software.
In the present study, simple solutions, which can be used on simple structures and demand little or no use of computer, are presented.

2. INTRODUCTION

Under creep in structures one understands time-dependent changes of strain and stress states taking place as a consequence of external loading and temperature.
Both the microstructural and the phenomenological description of creep can be applied. For the most conventional engineering applications, the phenomenological description is enough, i.e. the local changes in the structure of the material can be neglected and the mathematical formulation describing the phenomenon can be based on macroscopic observations. An historical review of the study of creep is given by H. Altenbach [2], J. Hult [3], J. Finnie and W. Heller [4]. It is obvious that the effect of creep on structures is of major concern for the mechanical engineers. Creep became of great interest for civil engineers due to the development of fire engineering. However, the design taking into account creep is generally different in mechanical and civil engineering applications. The time interval, in which the phenomenon is developed, is significantly lower not exceeding 2.5 hours in a typical fire. However, machine and machine elements are subjected to higher temperatures for longer time periods (10,000-100,000hrs.) and the developed stresses are usually lower than in civil engineering applications.

The past 20 years aluminium alloys structures are used in civil engineering applications due to a number of reasons: low weight and satisfactory mechanical strength, good corrosion resistance for certain alloys and good formability due to the extrusion process. As main disadvantages one could mention the low modulus of elasticity comparing to steel ($E_{alumin=}E_{steel}/3$) and the low melting point (600^{o}C). Eurocode prEN1999-1-2 [1] deals with the study of structures subjected to fire. According to this regulation, the explicit calculation of the deformations due to creep is necessary when the temperature exceeds 170^{o}C for over ½ hour. Design rules for steel structures subjected to fire are based on extensive experimental results and on plentiful experience. On the contrary, in the case of aluminium, only a few experimental results are available. An important contribution to the study of aluminium structures subjected to fire is the work of N.K. Langhelle [5], in which an extended bibliography is presented.

3. CREEP STRESS ANALYSIS

On the typical deformation-time creep curve, depicted in Fig. 1, three different regions corresponding to three different stages of creep can be observed:

1. Primary creep, in which the creep rate is decreasing
2. Secondary or steady state creep, in which the creep rate is steady
3. Tertiary creep is the final stage before fracture and has an increasing creep rate.

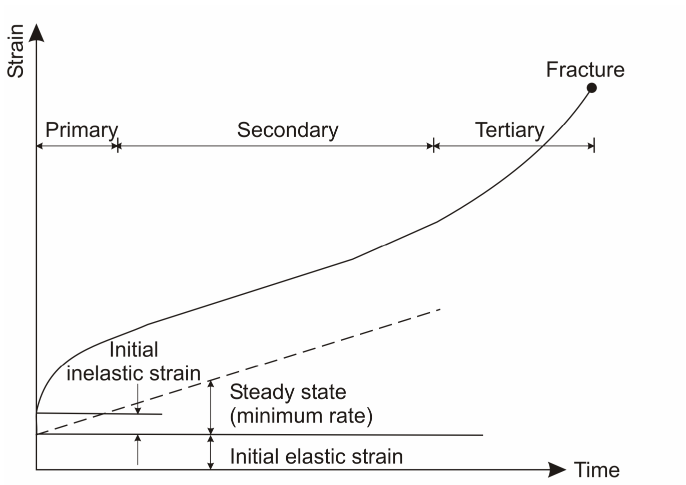

Fig. 1 Typical creep curve

It is generally accepted that in metals and in their alloys, the deformations, which interest the engineer, are developed in temperature $T \leq 0.4T_m$, where T_m is the melting point of the metal in ^{o}K (76.2^oC or 349^oK for aluminium alloys). According to Sandstrom [6], creep effect on aluminium alloy 6082 is important for engineering applications in temperatures above 75^{o}C.

The creep stress analysis is, in general, a cumbersome problem due to the nonlinearity of the solving differential equations. Except for a few cases (J. T. Boyle and J. Spence, [7]), creep problems do not lead to closed form equations. In statically indeterminate structures, a redistribution of stresses due to creep, takes place. This redistribution, known as transient creep, begins at t=0, (stresses are elastic) and continues until the distribution of stresses reach a steady state. During this procedure, higher elastic stresses tend to be decreased, whereas lower stresses tend to be increased. This procedure takes place in a relatively small time period and the developed deformations are rather small (Fig. 2). Then, to a good approximation [7], the superposition principle can be used. According to this, the deformations can be calculated by the superposition of en elastic part, as if no creep is present and a pure creep part, where no elastic deflections are present. Thereafter, the explicit calculation of deformations becomes unnecessary. Accordingly, the strain rate is given by the following equation:

$$\frac{d\varepsilon}{dt}=\frac{1}{E}\frac{d\sigma}{dt}+g(t)\sigma^{n} \qquad (1)$$

where:
g(t) is the time function and
n is a temperature dependent constant

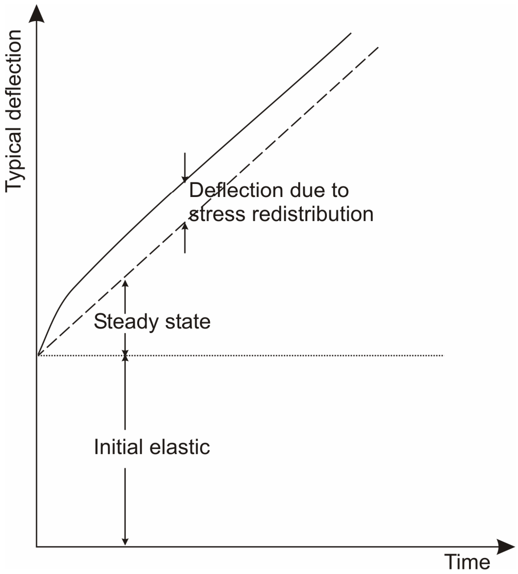

Fig. 2: Time variation of a typical displacement in a creeping structure under constant load

In statically determinate structures under steady temperature subjected to a constant load, the calculation of deflections in steady state creep derives from closed form equations. During this stage, stress and deformation distribution at the cross section remains stable and can be defined as a typical problem of elasticity, not taking into account the time parameter (elastic analogy). In steady creep of a structure, stresses and strain rates remain constant over time and the strains due to creep are given by:

$$\varepsilon = B\cdot\sigma^{n}\cdot t \qquad (2)$$

It can be said that in steady creep the curve in Fig. 1 is replaced by a line, which is parallel to the line segment of the secondary creep curve and the eq. (1) becomes accordingly:

$$\frac{d\varepsilon}{dt}=\frac{1}{E}\frac{d\sigma}{dt}+B\sigma^{n} \qquad (3)$$

Obviously, the major problem is the determination of B and n parameters, which are determined experimentally. It should be mentioned the lack of experimental results concerning the determination of these parameters in aluminium structures. In table 1, these parameter values for alloy EN AW 6082-T6 are given (N. K Langhelle, [5]).

Temperature	B	n	m
200^{o}C	$7.47\cdot10^{-12}$	2.31	-0.1
250^{o}C	$2.57\cdot10^{-12}$	15.06	-0.1

Table 1: B, n, m parameters

Langhelle has used the constitutive law of the following form:

$$\varepsilon = B \cdot \sigma^n \frac{t^{m+1}}{m+1} \qquad (4)$$

Approximately, this law can be substituted by a relation of the form given in eq. (2). J.G. Kaufman [8] gives also tabular data concerning other aluminium alloys, where the stresses corresponding to the relevant strain rates for certain time and temperature conditions, are given. Data for aluminium alloy EN AW 6061-T6 is given in Table 2.

Stress at 205^{o}C (MPa)	Time (hrs)	Strain rate
130	100	0,1%
95	1000	0,1%

Table 2: Stress and strain rates at 205^{o}C for 6061-T6 aluminium alloy for various times

Values of B= $3.04 \cdot 10^{-19}$ and n=7.34 can be derived with the appropriate fitting of eq. (2) to the data given in table 2.These values of B and n, perhaps are not considered to be accurate enough. However, it must be remembered that we are not dealing with elastic stresses, where the elastic constants and even the yield point of a given material can be measured with reasonable accuracy. In creep, small changes in such variables as stress, temperature, composition, heat treatment or method of manufacture may greatly influence creep behaviour and hence a simple empirical expression for the creep data may be adequate for rational design [4].

4. DEFORMATIONS OF A SIMPLE BEAM

For the calculation of the deflections of a beam in the steady state creep, the dummy force method can be used. This method is based upon the principle of the virtual work, which is valid in all cases, irrespective of the constitutive law.

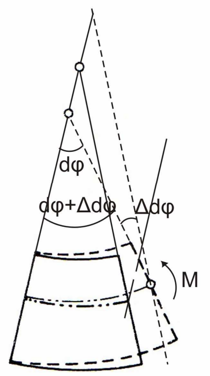

Fig. 3 Deformed shape of an infinitesimal element under bending

From Fig. 3 and considering that δ_{ip} is the value of deflection in point i due to the loading p, $\bar{M}(s)$ is the moment diagram derived using the unit dummy-force method and $\Delta d\varphi$ is the curvature of the infinitesimal element ds, it is derived that:

$$\Delta d\varphi = (\varepsilon/(h/2))ds \qquad (4)$$

where h is the height of the cross section. Substituting ε from eq. [2] gives:

$$\Delta d\varphi = \frac{B \cdot \sigma^n \cdot t}{h/2} ds = B\left(\frac{M(s)}{I_c} \cdot h/2\right)^n \cdot \frac{1}{h/2} \cdot ds \cdot t \qquad (5)$$

Accordingly, the deflection is given by:

$$\delta_{ip} = t \cdot \int_0^L \bar{M}(s) \cdot B\left(\frac{M(s)}{I_c} \cdot h/2\right)^n \cdot \frac{1}{h/2} \cdot ds \qquad (6)$$

In the appendix, the appropriate relations for the calculation of beam deformations with simple supports and loadings, are given.

In Eurocode EN 1999-1-2 the calculation of deflections is required, in order to eliminate the danger of failure in fire protective insulation and compartment walls. Despite this, no deflection limits are given. In BS 5950 Part 8 [9] is stated that "where a fire resisting wall liable to be subjected to significant additional vertical load due to the increased vertical deflection of a steel beam in a fire either:

(a)provision should made to accommodate the anticipated vertical movement of the beam or,
(b)the wall should be designed to resist the additional vertical load in fire conditions". R.M.Lawson and G.M.Newman in [10] give a deflection limit equal to L/200, where L is the span length. This provision could also be used in aluminium structures.

5. NUMERICAL APPLICATION

In the following, the creep deflection of a simple supported beam with length 3000mm, loaded by a concentrated force in the middle, is calculated. The cross-section of the beam, which is non-fire protected, is double T. The material is the heat-treated aluminium alloy EN AW 6082-T6. The beam is assumed to have a temperature of 200^{o}C with a duration of 3600 seconds. The parameter values B and n are taken from Langhelle results.

Geometry of the C/S	
h	114
b	120
t_w	4.6
t_f	11
All units in mm	

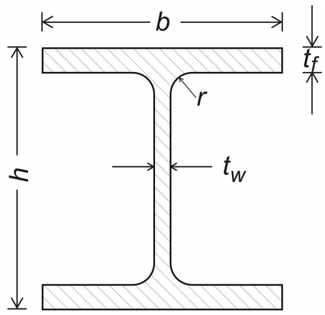

Fig. 4 Geometry of the cross section

The deflection of this beam according to the simple beam theory without taking into account the creep phenomenon is:

$$z = \frac{P \cdot l^3}{48 \cdot E \cdot I} = \frac{14000 \cdot 3000^3}{48 \cdot 70000 \cdot 748 \cdot 10^4} = 15,00\text{mm} \qquad (7)$$

From relation a in appendix, the following is derived:

$$z_{max} = \frac{2Bt}{h}\left(\frac{h}{2I_c}\right)^n \left(\frac{PL}{4}\right)^n \frac{L^2}{4(n+2)} =$$

$$=\frac{2\cdot 7,47E-12\cdot 3600}{114}\left(\frac{114}{2\cdot 748,2\cdot 10^{4}}\right)^{2,31}\left(\frac{14000\cdot 3000}{4}\right)^{2,31}\left(\frac{3000^{2}}{4\left(2,31+2\right)}\right)=6{,}14\text{mm} \qquad (8)$$

6. CONCLUDING REMARKS

In the present study, closed-form solutions for the calculation of creep deflections in a fire situation were presented. For the cases, where no closed-form solutions were proposed, the dummy force method, appropriately applied, can be used. Experimental data for different aluminium alloys, found in bibliography, were also used in an appropriate fitting of the power law describing the dependence of creep rate on stress.
From the given example was derived that creep deflection is 29% of the total deflection of a non-fire-protected beam with a concentrated loading in the middle subjected to a temperature of 200^{o}C. It should be remembered that the creep behaviour of various aluminium alloys is different and further research is needed.

7. APPENDIX

Calculation of deflection
1. Simple beam
a. Concentrated force in the middle P

$$z_{max}=\frac{2Bt}{h}(\frac{h}{2I_c})^n(\frac{PL}{4})^n\frac{L^2}{4(n+2)}$$

b. Distributed loading q

$$z_{max}=\int[z_0{}'-K\cdot\int_0^x|(X\cdot L-X^2)^n|dX]dx$$

where:

$$z_0{}'=K\cdot\int_0^{L/2}\left|(L\cdot x-x^2)^n\right|dx$$

$$K=\frac{2B\cdot t}{h}(\frac{h}{2I_c})^n(\frac{q}{2})^n$$

Also can be used the following closed-form solution proposed by F. Odqvist [11] for double T cross sections with a negligible stiffness of the web and an integer n.

$$z_{max}=\frac{4}{\pi^2}\cdot\frac{L^2}{2h}[\frac{qh^2}{8Ah}]^n\cdot B\cdot C_1\cdot t$$

C_1 is a constant , which is given in the following table for integer values of n from 1 to 7.

n	**C_1**
1	1,032049104
2	0,891088551
3	0,79576768
4	0,7257593
5	0,671527
6	0,62836
7	0,6258

2. Cantilever (I. Finnie,W.R.Heller, [4])

a. Distributed loading q

$$z_{max} = \frac{2Bt}{h}\left(\frac{h}{2I_c}\right)^n\left(\frac{qL^2}{2}\right)^n\frac{L^2}{2(n+1)}$$

where:

h height of the cross section
B, n creep parameters
A area of the flange of a double T cross section
L length of the beam
I_c virtual moment of inertia, which is:
 3nI/(2n+1) for rectangular cross sections and
 $I_c \approx I$, in double T cross sections, when the web has negligible stiffness
t time

8. REFERENCES

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Ο ερπυσμός σε κατασκευές από αλουμίνιο

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Η εργασία αφορά στη μελέτη της επίδρασης του ερπυσμού σε δομικά στοιχεία από κράματα αλουμινίου, που εκτίθενται σε πυρκαγιά. Σύμφωνα με το ισχύον κανονιστικό πλαίσιο (prEN1999-1-2:2006), ο αναλυτικός υπολογισμός των βυθίσεων λόγω ερπυσμού σε δομικά στοιχεία από αλουμίνιο, επιβάλλεται στην περίπτωση, που η θερμοκρασία του δομικού στοιχείου υπερβαίνει τους 170^{o} C για χρονικό διάστημα μεγαλύτερο της μισής ώρας. Γενικά ο υπολογισμός των βυθίσεων δομικών στοιχείων λόγω ερπυσμού με την ανάπτυξη πυρκαγιάς απαιτεί τη χρήση εξειδικευμένου λογισμικού. Ο υπολογισμός των μετακινήσεων λόγω ερπυσμού βασίζεται σε αρχές της μηχανικής (επαλληλία καταστάσεων, ελαστικό ανάλογο, μέθοδος πλασματικής δύναμης). Οι αρχές αυτές σε συνδυασμό με πειραματικά δεδομένα έχουν ευρέως χρησιμοποιηθεί κατά τη μελέτη κατασκευών από χάλυβα, όπου το φαινόμενο έχει μελετηθεί εκτεταμένα. Στα κράματα αλουμινίου, σε αντιδιαστολή με το χάλυβα, υπάρχει σημαντική έλλειψη πειραματικών αποτελεσμάτων, η οποία σε συνδυασμό με την ποικιλότητα, που εμφανίζουν αυτά, όσο αφορά στις μηχανικές τους ιδιότητες (συμβατικό όριο διαρροής $f_{0.2}$ και όριο θραύσης f_u) επιτείνει τις δυσκολίες αντιμετώπισης του ερπυσμού σε αυτά. Στην παρούσα εργασία παρουσιάζονται κλειστού τύπου λύσεις για τον υπολογισμό των βυθίσεων, σε τυπικά καμπτόμενα δομικά στοιχεία υπό διάφορες συνθήκες φόρτισης. Στις λύσεις αυτές χρησιμοποιούνται συντελεστές για τον ερπυσμό, οι οποίοι λαμβάνονται από πειραματικά δεδομένα, που έχουν βρεθεί στη βιβλιογραφία για τα θερμικώς κατεργαζόμενα κράματα EN AW 6061-T6 και EN AW 6082-T6, τα οποία κυρίως χρησιμοποιούνται στις δομικές εφαρμογές. Οι, κλειστού τύπου, αυτές λύσεις αποτελούν ένα χρήσιμο εργαλείο καθώς προσφέρουν μία επαρκή αντιμετώπιση του προβλήματος χωρίς τη χρήση ειδικού λογισμικού.

Στην εργασία αυτή εκτίθεται τέλος ένα παράδειγμα υπολογισμού των βυθίσεων λόγω ερπυσμού μίας αμφιέρειστης δοκού με φορτίο στο μέσο χωρίς πυροπροστατευτική μόνωση για το κράμα EN AW 6082T6.