

# EVALUATING THE EPISTEMIC UNCERTAINTY OF THE SEISMIC DEMAND AND CAPACITY FOR A 9-STORY STEEL MOMENT-RESISTING FRAME

**Dimitrios Vamvatsikos, Ph.D.**

*School of Civil Engineering*

*National Technical University of Athens, Greece*

E-mail: [divamva@mail.ntua.gr](mailto:divamva@mail.ntua.gr)

**Dimitrios G. Lignos, Ph.D.**

*Department of Civil Engineering and Applied Mechanics*

*McGill University, Montreal, Canada*

E-mail: [dimitrios.lignos@mcgill.ca](mailto:dimitrios.lignos@mcgill.ca)

## 1. ABSTRACT

The accurate estimation of the seismic performance of steel structures requires reliable information on the effect of our incomplete knowledge of the actual system parameters. Aiming to provide such an outlook we undertake a comprehensive effort to quantify the uncertainty for a single steel moment-resisting frame by bringing together several important advances. Model parameters are described by complete probabilistic distributions including intra-member and inter-member correlation information derived from experimental data from a recently developed database for modeling steel components. Incremental dynamic analysis is employed to accurately assess the seismic performance of the model for any combination of the parameters by performing multiple nonlinear timehistory analyses for a suite of ground motion records. Finally, we use an efficient Monte Carlo simulation algorithm based on incremental Latin Hypercube Sampling to efficiently propagate the uncertainties from the numerous parameters to the actual system demand and capacity. The effect of model parameter uncertainties on the seismic behavior of the 9-story steel moment resisting frame is thus quantified, offering a unique method to assess the actual margin of safety inherent in any steel frame structure.

## 2. INTRODUCTION

The evaluation of the seismic demand and capacity of structures stands at the core of performance-based earthquake engineering. While guidelines have emerged [1] that recognize the need for assessing epistemic uncertainties by explicitly including them in estimates of seismic performance, this role is usually left to ad hoc safety factors, or, at best, standardized dispersion values that often serve as placeholders. Still, seismic performance is heavily influenced by both aleatory randomness, e.g. due to natural ground motion record variability, and epistemic uncertainty, owing to modeling assumptions, omissions or errors. While the first can be easily estimated by analyzing a given structure

under multiple ground motion records, e.g via incremental dynamic analysis (IDA, [2]), estimating the epistemic uncertainty remains a little-explored issue.

Recently, several researchers have proposed applying nonlinear dynamic analysis combined with Monte Carlo simulation to quantify the uncertainty for structural models with non-deterministic parameters. For example, Ibarra [3] actually proposes a method to propagate the uncertainty from model parameters to structural behavior using first-order-second-moment (FOSM) principles verified through Monte Carlo to evaluate the collapse capacity uncertainty. As a performance improvement, Latin Hypercube Sampling (LHS) [4] has also been proposed instead of classic random sampling. Kazantzi et al. [5] used Monte Carlo with LHS to incorporate uncertainty into steel frame fragility curves. Liel et al. [6] used IDA with Monte Carlo and FOSM coupled with a response surface approximation method to evaluate the collapse uncertainty of several reinforced-concrete buildings. On a similar track, Dolsek [7] and Vamvatsikos & Fragiadakis [8] have proposed using Monte Carlo with efficient LHS on IDA to achieve the same goal.

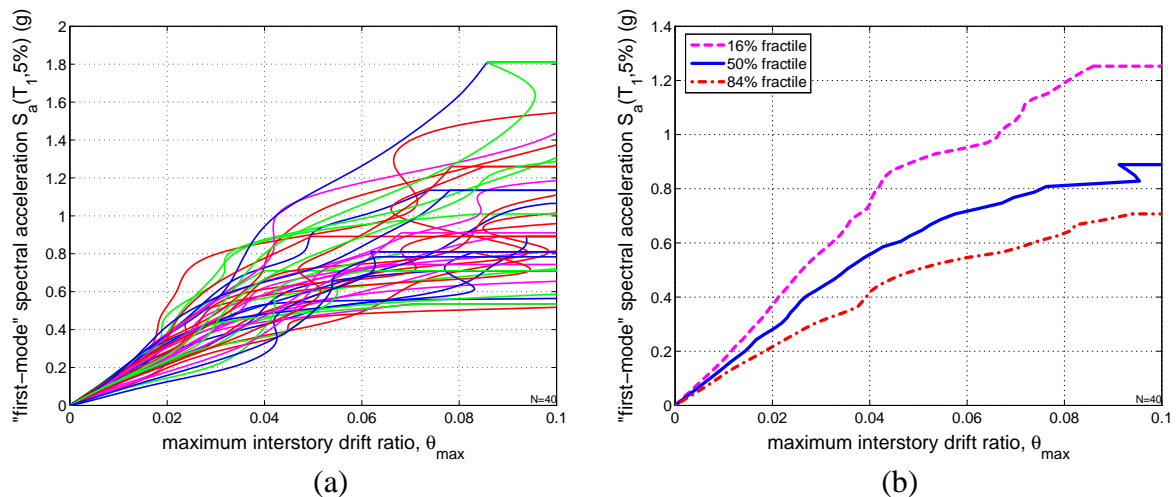
However, any practical application of the above LHS-based methodologies is severely restricted due to two important reasons. The first is our inherent inability to determine in advance the required number of observations. Due to the nature of LHS, the entire sample has to be decided *a priori*. While for typical random sampling we can stop the simulation at will, before examining the entire sample, doing so for LHS is not possible unless we want to risk a biased estimate. Similarly, if after the end of the simulation we realize that we need more observations, we cannot easily reuse the existing ones by arbitrarily adding to them; the end product will typically not be a proper latin hypercube design. In other words, we are limited by our initial knowledge of the problem to be able to select a proper sample size, which may or may not be correct on the first try and often leads to repeated analyses. The second reason is the disproportionate increase in the number of analyses when dealing with many random variables. It may become prohibitively expensive to determine the influence of multiple random parameters, as the sample size rises disproportionately. This is what has lead all early attempts [6–8] to limit themselves to just a handful of parameters.

To overcome these important limitations, we will reorganize the application of Monte Carlo with LHS on IDA by performing together the model and record sampling and using incremental sample sizes that have been carefully selected to allow full reuse of the earlier runs performed. Thus, we propose an efficient upgrade to the original approach that is applicable to large models with hundreds of random variables and without any need of pre-determining sample sizes in any way.

### 3. INCREMENTAL DYNAMIC ANALYSIS

Incremental Dynamic Analysis (IDA) is a powerful analysis method that offers thorough seismic demand and capacity prediction capability [2]. It involves performing a series of nonlinear dynamic analyses under a multiply-scaled suite of ground motion records, selecting proper Engineering Demand Parameters (EDPs) to characterize the structural response and an Intensity Measure (IM), e.g. the 5%-damped first-mode spectral acceleration,  $S_a(T_1, 5\%)$ , to represent the seismic intensity. The results are presented as curves of EDP versus IM for each record (*Fig. 1a*). These can be further summarized into the 16,50,84% fractile IDA curves by estimating the respective percentile values given a

range of IM or EDP values. Appropriate limit-states can be defined by setting limits on the EDPs. The probabilistic distribution of limit-state capacities can be easily estimated, e.g. for limiting values of the maximum interstory drift by reading off the median and the dispersion of the required  $S_a$  capacity from *Fig. 1b*. Such results combined with probabilistic seismic hazard analysis [2] allow the estimation of mean annual frequencies (MAFs) of exceeding the limit-states, thus offering a direct characterization of seismic performance. Nevertheless, IDA comes at a considerable cost, even for simple structures, necessitating the use of multiple nonlinear dynamic analyses that are usually beyond the abilities and the computational resources of the average practicing engineer. Therefore, wherever IDA is involved, searching for an efficient implementation is always desirable.



*Fig. 1 (a) Forty IDA curves and (b) their summarization into 16,50,84% fractile curves*

#### 4. INCREMENTAL RECORDWISE LHS

To mitigate the issues related to the typical application of LHS on IDA, we propose using the same two fundamental procedures but essentially redefine the way that they are implemented by incorporating two important changes. First, latin hypercube sampling is applied incrementally by starting with a small sample that is doubled successively until adequate accuracy has been achieved. This is perhaps the only way that one can reuse the results of a previous LHS design, since doubling the size allows a simple way to insert new observations within the existing sample while maintaining all the properties and advantages of LHS. Thus, by comparing the convergence of the IDA results in successive generations of the LHS design, the development of a rational stopping rule becomes possible. This essentially offers an intuitive way to determine a reasonable sample size, minimizing the waste of runs over repeated tries or the (equally wasteful) tendency to overestimate the size to “get it right” in one step. Actually, the proposed amendment is simple enough that it has probably already appeared in the literature although the authors have not been able to find a relevant publication yet. Still, the use of LHS is so extensive that it is reasonable to surmise that something similar must have already appeared.

Furthermore, by taking advantage of the fact that IDA is itself a sampling process at equiprobable points (or records), we propose that LHS is performed simultaneously on the structural properties and on the ground motion records. Instead of maintaining the same properties for a given model realization over an entire ground-motion record suite, model parameter sampling is performed on a record-by-record basis, efficiently expanding the

number of observations without increasing the number of nonlinear dynamic analyses. As a further bonus, the incident angle of the record may also be varied to allow for including its effect as well. If we need more observations than the records available, the records can be simply recycled, either with the same or a different incident angle. In the customary application of such a procedure, each model realization would be subject to IDA for the entire record suite, multiplying the number of nonlinear dynamic analyses by a factor of 20 – 60. By combining the innovations presented we have formed iLHS, an efficient algorithm that is applicable to large models with hundreds of random variables [9].

## 5. EXAMPLE APPLICATION

### 5.1 Model description

The structure selected is a nine-story steel moment-resisting frame with a single-story basement that has been designed for Los Angeles, following the 1997 NEHRP (National Earthquake Hazard Reduction Program) provisions [11]. A centerline model with nonlinear beam-column connections was formed using OpenSees. It allows for plastic hinge formation at the beam ends while the columns are assumed to remain elastic. The structural model also includes P- $\Delta$  effects while the internal gravity frames have been directly incorporated. The fundamental period of the reference frame is  $T_1 = 2,35s$  and accounts for approximately 84% of the total mass. Essentially this is a first-mode dominated structure that still allows for some sensitivity to higher modes.

The beam-hinges are modeled as rotational springs with a quadrilinear moment-rotation backbone (*Fig. 2*) based on the modified Ibarra-Krawinkler deterioration model [3,10]. The backbone curve of this model (see *Fig. 2a*) is defined based on the elastic stiffness  $K_e$  of the component, its pre-capping plastic rotation  $\theta_p$  the post-capping plastic rotation  $\theta_{pc}$  a residual strength  $M_r$  that is expressed as a function of the yield strength  $M_y$  and an ultimate rotation capacity  $\theta_r$ . The same model is able to simulate up to 4 modes of cyclic deterioration (see *Fig. 2b*). Lignos and Krawinkler [11] calibrated the hysteretic response of this model with the deduced moment rotation relationship of more than 300 tests included in a database for deterioration modeling of steel components. The deterioration model parameters are estimated based on multivariate regression relationships. *Fig. 3* shows the cumulative distribution functions of  $\theta_p$  and  $\theta_{pc}$  for steel beams with and without Reduced Beam Sections (RBS). Lognormal distributions were found to fit the experimental data relatively well based on a Kolmogorov-Smirnov test.

In order to evaluate the effect of uncertainties on the seismic performance of the structure we chose to vary the beam-hinge backbones according to the fitted probabilistic distributions of their parameters, as shown in *Fig. 3*. The hinges at the end of each individual beam were assumed to be perfectly correlated. *Table 1* summarizes the correlations between deterioration parameters assumed in this study. Note that these correlations are based on actual experimental data as discussed earlier. Within the same beam, all parameters are independent except the rotations  $\theta_p$  and  $\theta_{pc}$  that share an 80% correlation coefficient. Among different beams in any given story, a 70% correlation was employed for each parameter. Among beams in different stories only 50% correlation was used.

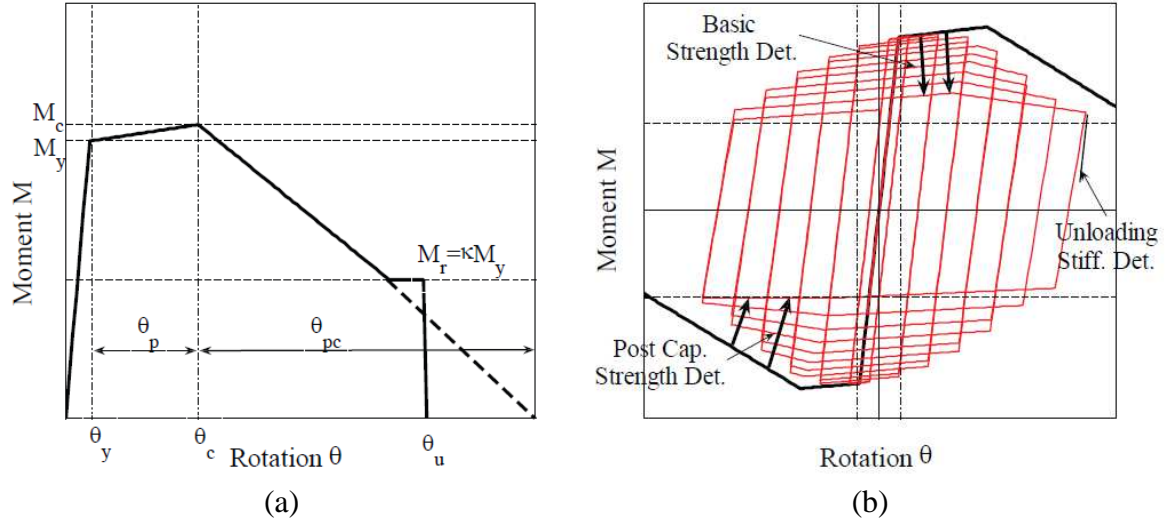


Fig. 2 Monotonic and cyclic moment rotation relationship of the modified Ibarra-Krawinkler deterioration model [3,10]

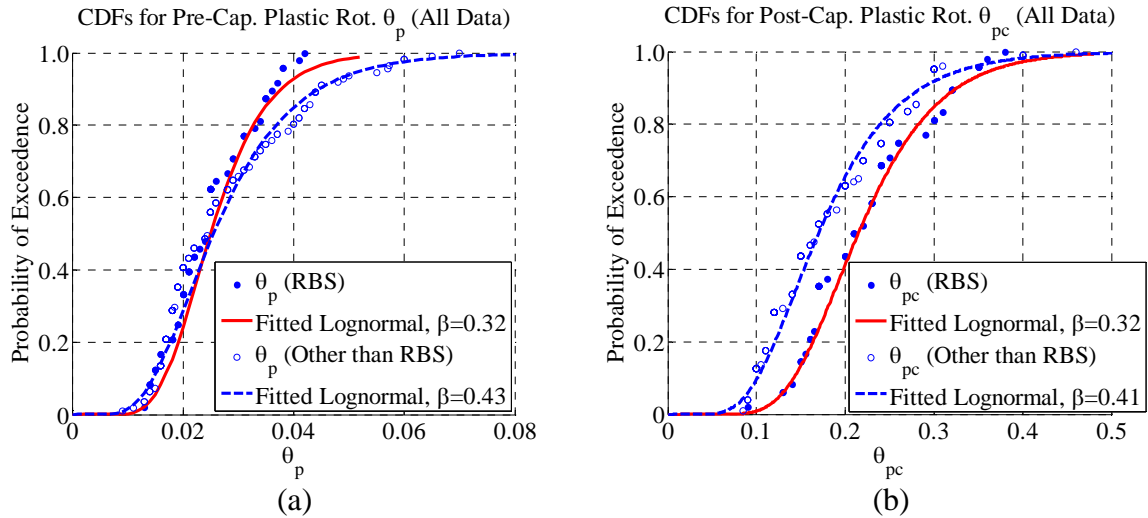


Fig. 3 Cumulative distribution functions (CDFs) for (a)  $\theta_p$  and (b)  $\theta_{pc}$  [10]

Plastic Rotations	Pre-cap $\theta_p$	Post-cap $\theta_{pc}$	Cumulative $\Lambda$
Pre-cap $\theta_p$	1	0,69	0,44
Post-cap $\theta_{pc}$	0,69	1	0,67
Cumulative $\Lambda$	0,44	0,67	1

Table 1. Correlation coefficients for deterioration parameters of non-RBS steel beams [13]

## 5.2 Illustrative results

Having a total of 270 random variables and 60 “ordinary” ground motion records (i.e. without any soft soil or directivity issues), the iLHS algorithm is applied with a starting size of 10 and it is allowed to run to a total of 8 generations, up to a maximum sample size of 1280. There are  $270 \cdot (270-1)/2 = 36315$  correlation coefficients to match, thus some error is bound to appear when having only a few observations. Thanks to the algorithm used to

impose correlation [14], this error can be selectively minimized for any variables that are deemed to be important. Therefore, correlation is nearly perfectly captured for the most influential variables, i.e.  $M_y$  and  $\theta_p$ , leading to an overall accurate estimation.

The simulation was run in parallel [15] using 5 Pentium IV single-core processors for an overall running time of 10hrs. Actually the 1280 observations are far too many. The median and the dispersion  $\beta$  (standard deviation of the log of the data) of  $S_a$  to achieve a certain response value become fairly stable for practically all EDPs after only 4–5 generations with 160–320 samples, respectively. The results seem to only mildly differ among the global or local EDPs considered, e.g. the roof drift  $\theta_{\text{roof}}$  and maximum interstory drift  $\theta_{\text{max}}$ , or the individual  $i$ -story drifts  $\theta_i$ .

In comparison to a typical analysis considering only the mean-parameter model, the dispersion is found to be similar (Fig. 4b) but the mean response itself has a prominent bias, which, due to the details of the correlation imposed, appears to be a conservative one (Fig. 4a). It is also possible to determine the influence of each random variable by measuring its correlation with the estimated response values [7]. Then, we find that, at least for  $\theta_{\text{max}}$ , the most influential variables in the lower, near-yield, limit-states ( $\theta_{\text{max}} = 0,02$ ) mainly involve the  $\theta_p$  and  $\theta_{pc}$  variables at a meager 8–10% correlation. For higher limit-states, closer to collapse, we slowly start to see the effect of the yield strength in the middle stories, with the correlation progressively rising up to the order of 22%. Such information can be extracted to any detail and for each response type and structural state desired.

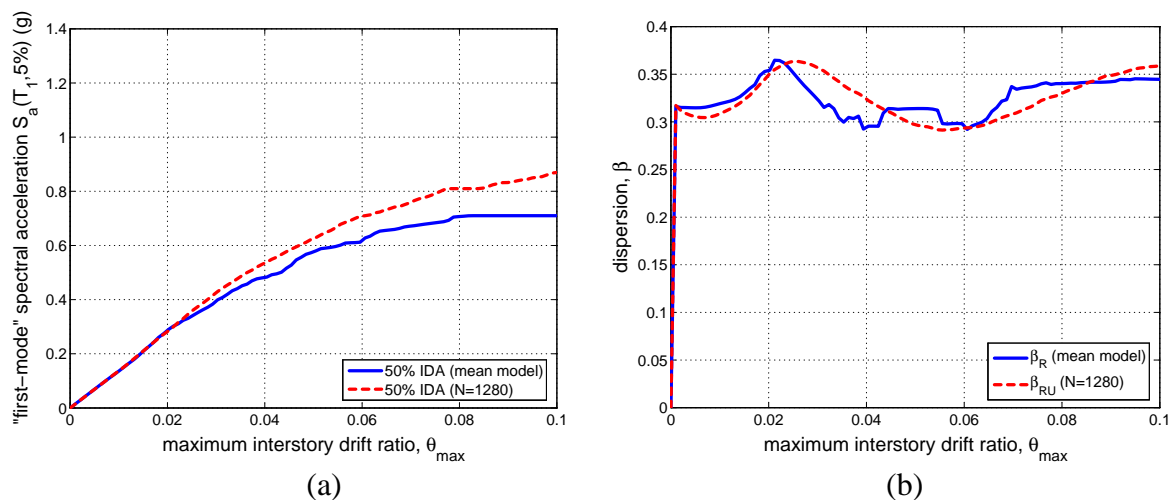


Fig. 4 iLHS versus the mean model: (a) comparison of median IDAs and (b) comparison of  $S_a$ -capacity dispersion

## 6. CONCLUSIONS

An efficient method has been presented for accurately determining the seismic demand and capacity uncertainty of steel moment-resisting frames by combining realistic probabilistic modeling together with innovative analysis and sampling techniques. The model sample is formed using appropriate deterioration parameters for steel components based on a recently developed database of experimental results. Processing is based on the incremental Latin Hypercube Sampling procedure that is capable of efficiently estimating the effect of model parameter uncertainties on the seismic performance of structures. It

builds upon the existing paradigm of incremental dynamic analysis with latin hypercube sampling and further improves it by resolving the problem of sample size determination and by increasing its efficiency by a factor of 20 at least. It is a simple technique that is amenable to parallelization and automated application while it allows excellent scalability, being applicable to realistic large-scale problems. The methodology presented can thus estimate the seismic performance uncertainty of steel moment-resisting frames and provide reliable values where formerly only mere placeholders were available.

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## **ΕΚΤΙΜΗΣΗ ΤΗΣ ΕΠΙΣΤΗΜΙΚΗΣ ΑΒΕΒΑΙΟΤΗΤΑΣ ΣΤΗ ΣΕΙΣΜΙΚΗ ΑΠΟΚΡΙΣΗ ΚΑΙ ΙΚΑΝΟΤΗΤΑ 9-ΩΡΟΦΟΥ ΜΕΤΑΛΛΙΚΟΥ ΠΛΑΙΣΙΟΥ**

**Δημήτριος Βαμβάτσικος**  
*Σχολή Πολιτικών Μηχανικών*  
*Εθνικό Μετσόβιο Πολυτεχνείο, Αθήνα*  
E-mail: [divamva@mail.ntua.gr](mailto:divamva@mail.ntua.gr)

**Δημήτριος Γ. Λιγνός**  
*Τμήμα Πολιτικών Μηχανικών και Εφαρμοσμένης Μηχανικής*  
*Πανεπιστήμιο McGill, Μοντρεάλ, Καναδάς*  
E-mail: [dimitrios.lignos@mcgill.ca](mailto:dimitrios.lignos@mcgill.ca)

Η ακριβής εκτίμηση της σεισμικής επιτελεστικότητας των μεταλλικών κατασκευών απαιτεί αξιόπιστη πληροφόρηση σχετικά με την επιρροή της ατελούς γνώσης των πραγματικών δομικών παραμέτρων. Με αυτό το σκοπό, επιχειρείται η ποσοτικοποίηση της αβεβαιότητας για ένα μεταλλικό πλαίσιακό κτίριο με το συνδυασμό μιας σειράς καινοτομιών. Καταρχάς, οι βασικές παράμετροι του προσομοιώματος περιγράφονται πλήρως πιθανοτικά, συμπεριλαμβανομένης της συσχέτιση εντός κάθε μέλους, αλλά και μεταξύ των μελών, σύμφωνα με μια πρόσφατα ανεπτυγμένη βάση πειραματικών δεδομένων για την προσομοίωση μεταλλικών στοιχείων. Η ανάλυση δυναμικής αντίστασης (IDA) εφαρμόζεται, για να εκτιμηθεί με ακρίβεια η συμπεριφορά του προσομοιώματος σε κάθε συνδυασμό των παραμέτρων με την εκτέλεση μη γραμμικών δυναμικών αναλύσεων υπό πολλαπλά επιταχυνσιογραφήματα. Χρησιμοποιείται ένας αποδοτικός αλγόριθμος Μόντε Κάρλο βασισμένος σε αυξηματική δειγματοληψία λατινικού υπερκύβου, για να μεταδοθούν οι αβεβαιότητες από το σύνολο των παραμέτρων στη σεισμική απαίτηση και ικανότητα. Κατά αυτόν τον τρόπο, επιτυγχάνεται η ποσοτικοποίηση της επίδρασης των αβέβαιων παραμέτρων του προσομοιώματος στη σεισμική συμπεριφορά του εννιαώροφου κτιρίου, παρέχοντας έτσι μια μοναδική μέθοδο εκτίμησης του πραγματικού διαθέσιμου περιθωρίου ασφάλειας που υπάρχει σε οποιαδήποτε μεταλλική πλαίσιακή κατασκευή.