BUCKLING DESIGN OF CONFINED STEEL CYLINDERS UNDER EXTERNAL PRESSURE

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1. ABSTRACT

Thin-walled steel cylinders surrounded by an elastic medium, when subjected to uniform external pressure may buckle. Using a two dimensional model with nonlinear finite elements, relevant design guidelines are developed, compatible with the general provisions of European design recommendations for shell buckling. Special emphasis is given on the deformability effects of the surrounding medium and imperfection sensitivity.

2. INTRODUCTION

In several applications, externally pressurized steel cylinders are confined within a rigid or deformable cavity [1]: buried steel pipelines steel liners for the rehabilitation of damaged pipelines, steel tunnels and ducts in power, and steel casing in oil and gas production wells. Due to the confining effect of the cavity, the steel cylinder cannot ovalize, and buckling occurs in the form of an "inward lobe" [2], as shown in Figure 1.

Considering a two-dimensional energy formulation of the deformed cylinder, Glock [3] resulted in a closed-form analytical solution for the ultimate pressure sustained by a cylinder of elastic material in a non-deformable (rigid) cavity:

$$p_{GL} = \frac{E}{1-\nu^2}\left(\frac{t}{D}\right)^{2.2} \qquad (1)$$

where E is Young's modulus, ν is Poisson's ratio, D is the cylinder diameter and t is the wall thickness. For D/t values between 100 and 300, the ultimate value of pressure p_{GL} calculated from equation (1) is 20 – 48 times higher than the buckling (bifurcation) pressure p_e of a long (free of boundary conditions) perfectly round elastic cylinder under unconfined conditions. On the other hand, the corresponding buckling problem of steel cylinders with elastic-plastic material, has received less attention. As a first approximation, the ultimate external pressure capacity can be estimated as the pressure that causes first yielding of the cylinder wall, a concept adopted by Montel [4], who proposed a semi-empirical formula for the buckling pressure of cylinders embedded in a stiff cavity, in terms of the material yield stress σ_y, the cylinder geometry D/t, the initial out-of-roundness with amplitude δ_0 and the initial gap with maximum value g between the cylinder and the rigid cavity:

$$p_M = \frac{14.1\,\sigma_y}{(D/t)^{1.5}\left[1+1.2(\delta_0+2g)/t\right]} \tag{2}$$

Equation (2) is valid for $60 \le D/t \le 340$, $250\text{ MPa} \le \sigma_y \le 500\text{ MPa}$, $0.1 \le \delta_0/t \le 0.5$, $g/t \le 0.25$ and $g/R \le 0.0025$. The value of p_M is well below the plastic pressure $p_y = 2.26\,\sigma_y\,(t/D)$ of the cylinder under plane strain conditions. Recently, Vasilikis & Karamanos [5] investigated thin-walled steel cylinders, surrounded by an elastic medium, in terms of their structural stability under uniform external pressure using a nonlinear two-dimensional finite element model, which accounted for large displacements and inelastic material behavior. They continued their work in [6] and proposed a simple and efficient design methodology, for the structural stability design of confined steel cylinders, within the framework of the new European shell stability design rules and recommendations [7] [8]. The design methodology is outlined in the present paper.

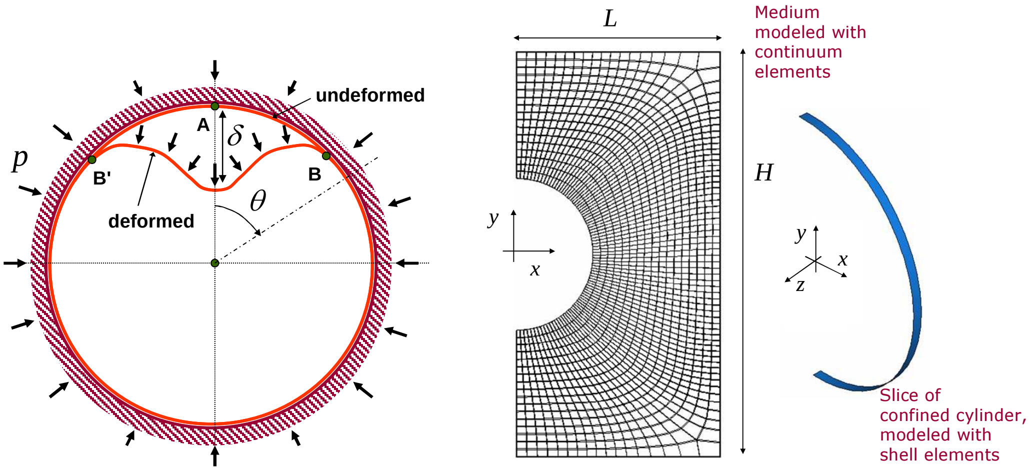


Figure 1. (a) Schematic representation of the buckling problem of an externally-pressurized confined cylinder in a cavity; (b) Finite element model of cylinder-medium system.

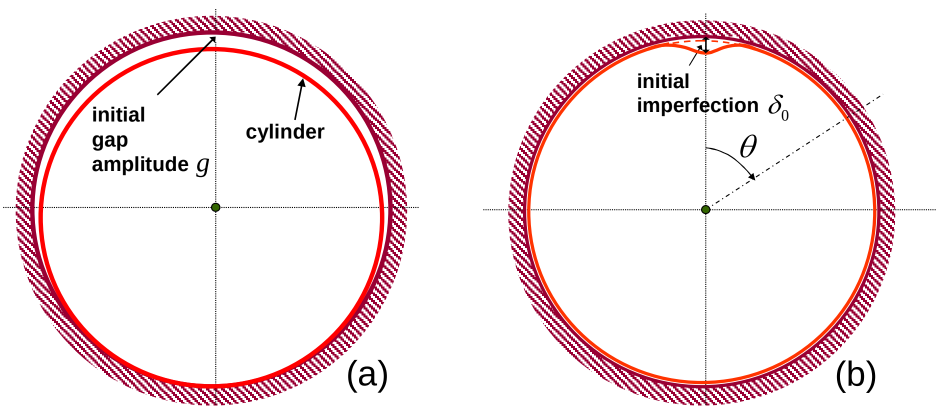


Figure 2. Schematic representation of a confined ring with (a) gap-type initial imperfection and (b) "out-of-roundness" initial imperfection.

3. FINITE ELEMENT MODELING

General-purpose finite element program ABAQUS is used, with nonlinear geometry and inelastic material behavior. No variation of loading and deformation is assumed along the cylinder, considering a two-dimensional finite element model of the cylinder with one element in the longitudinal direction of the cylinder, under plane-strain conditions. Using symmetry, half of the cylinder is analyzed, applying appropriate symmetry conditions at the $\theta = 0$ plane. The steel cylinder is modeled with four-node reduced-integration shell elements (type S4R), and eight-node reduced-integration solid elements (C3D8R) are used for the surrounding medium (Figure 1b). A frictionless contact algorithm is employed for the interface between the cylinder and the medium. Uniform external pressure is applied around the cylinder, and the pressure-deflection ($p-\delta$) equilibrium path is traced using a Riks continuation algorithm. Two types of initial imperfections are considered: (a) an initial gap between the confining medium and the cylinder and (b) a small initial "out-of-roundness" imperfection on the steel cylinder in the form of a small localized ("single-lobe") displacement pattern at the vicinity of the $\theta = 0$ location (Figure 2).

4. DESIGN METHODOLOGY FOR STEEL CYLINDERS

To describe buckling of confined cylinders under external pressure in a simple and efficient manner, a methodology is developed, based on "shell slenderness" [7] [8]:

$$\lambda = \sqrt{R_{pl}/R_{cr}} \quad (3)$$

where R_{pl} is the load at the plastic limit and R_{cr} the load corresponding to the elastic buckling condition of the perfect cylinder. The fully-plastic pressure p_y can be used for R_{pl}, whereas p_{GL} in equation (1) offers a very good prediction of R_{cr}, so that:

$$\lambda = \sqrt{\frac{p_y}{p_{GL}}} = \sqrt{\frac{2.26\,\sigma_y(1-\nu^2)}{E}\left(\frac{D}{t}\right)^{1.2}} \quad (4)$$

The numerical results of Figures 3 refer to cylinders with stiff confinement and three values of yield stress σ_y ($235\,MPa$, $313\,MPa$ and $566\,MPa$), showing the variation of ultimate pressure $p_{\max}$ in terms of λ. In the case of perfect cylinders ($g = \delta_0 = 0$), and for λ values greater than 2.2, buckling occurs in the elastic range, and the ultimate pressure $p_{\max}$ is equal to the one predicted by equation (1). Therefore,

$$\frac{p_{\max}}{p_y} = \frac{1}{\lambda^2}, \quad \text{for } \lambda \geq 2.2 \quad (5)$$

The value of 2.2 defines the transition between elastic and inelastic region, and it is denoted by λ_p. Furthermore, for imperfect cylinders, taking into account an imperfection reduction factor α, one can write

$$\frac{p_{\max}}{p_y} = \frac{\alpha}{\lambda^2}, \quad \text{for } \lambda \geq \lambda_p = 2.2 \quad (6)$$

In the present study, the reduction factor α, is assumed in the following form:

$$\alpha = C/\Delta^m \quad (7)$$

where Δ is an imperfection parameter that represents the size of the initial imperfection, considering both out-of-roundness and gap, and C, m are constant coefficients to be determined from the numerical results. The results in [5] [6] indicate a dependency of the imperfection sensitivity on the D/t value. Based on those results, this imperfection parameter is considered in the following form

$$\Delta = \left(\frac{\delta_0 + Kg}{R} \right) \sqrt{\left(\frac{D}{t} \right)} \qquad (8)$$

In the above expression, coefficient K expresses the relative influence of the two forms of imperfections (gap and out-of-roundness) on the ultimate pressure $p_{\max}$. From the numerical results, a value equal to 3 is obtained for this coefficient ($K = 3$). Upon determining the value of K, a standard curve fitting technique is employed, the values of C and m are calculated equal to 0.15 and 0.7 respectively, so that the elastic reduction factor becomes:

$$\alpha = \frac{0.15}{\Delta^{0.7}} = 0.15 \left[\left(\frac{\delta_0 + 3g}{R} \right) \sqrt{\left(\frac{D}{t} \right)} \right]^{-0.7} \qquad (9)$$

The imperfection reduction factor predicted through equation (9) is plotted against finite element results in Figure 4. The comparison indicates that equation (9) can provide good predictions for the ultimate pressure of externally pressurized elastic cylinders in the presence of initial imperfections.

For values of λ less than the plastic limit slenderness (λ_p =2.2), buckling occurs in the inelastic range, and the values of $p_{\max}$ for perfect cylinders deviate from the elastic solution (5). For very small values of λ, the value of $p_{\max}$ approaches the plastic pressure p_y. For the buckling pressure in the inelastic regime, the following expression, introduced in [7] [8] is adopted

$$\frac{p_{\max}}{p_y} = 1 - \beta \left(\frac{\lambda - \lambda_0}{\lambda_p - \lambda_0} \right)^{\eta}, \qquad \text{for } \lambda_0 \le \lambda \le \lambda_p \qquad (10)$$

where β is constant, η depends on imperfection parameter Δ and slenderness λ_0 is referred to as "squash limit relative slenderness"; for slenderness values less than λ_0, the cylinder collapses due to excessive plastification, so that neglecting strain hardening:

$$\frac{p_{\max}}{p_y} = 1, \qquad \text{for } \lambda \le \lambda_0 \qquad (11)$$

The numerical results of Figures 3 indicate that a value of λ_0 equal to 0.25 is representative of the cylinder behavior. Furthermore, in equation (10), the value of β is determined equating expressions (6) and (10) for $\lambda = \lambda_p$, and one readily obtains:

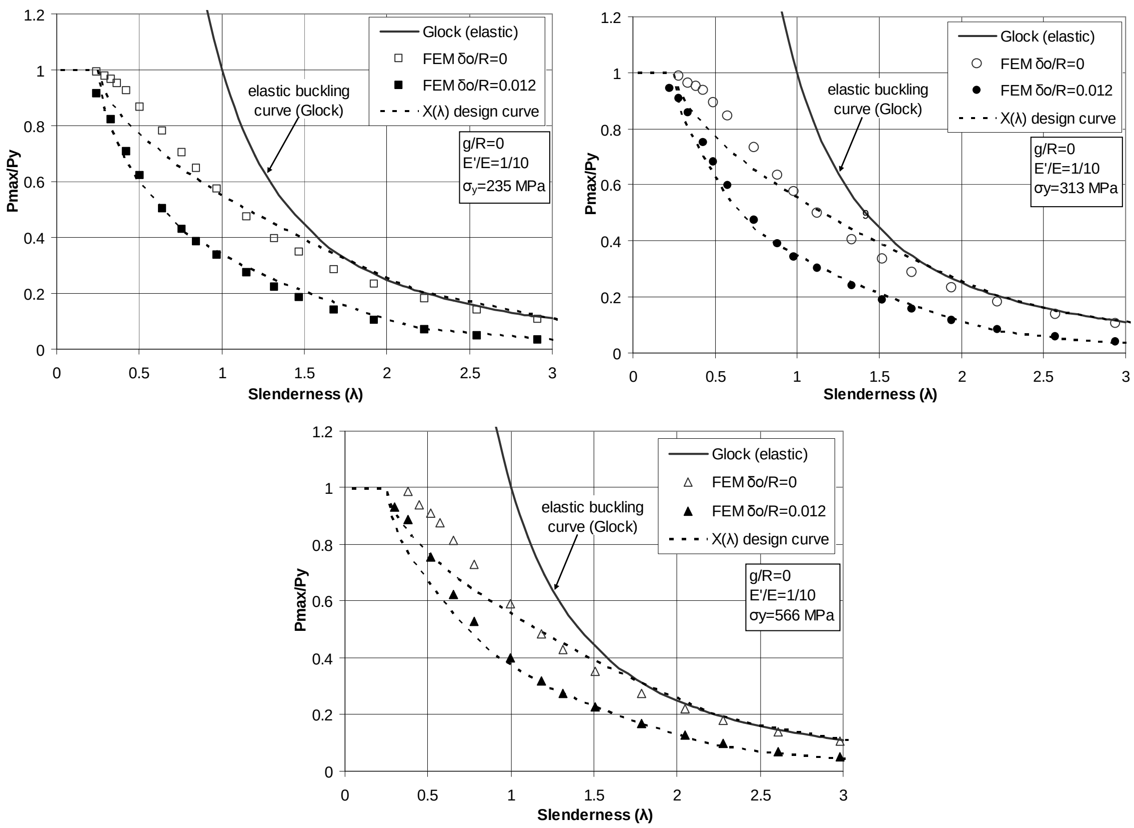
$$\beta = 1 - \frac{\alpha}{\lambda_p^2} \qquad (12)$$

The numerical results indicate a dependence of η on the initial imperfection, which can be expressed as follows:

$$\eta = 0.6 - 3\Delta, \; \eta \ge 0.3 \qquad (13)$$

Figures 3 show the predictions of the above design methodology for imperfect steel cylinders against the numerical finite element results. The comparisons indicate that the proposed methodology offers an efficient approach for predicting the ultimate pressure of confined cylinders in both the elastic and the inelastic range. The methodology is fully compatible with the general methodology for shell buckling design [7] [8].

As an alternative to the design methodology proposed in the previous paragraphs, Montel's equation can be used for the buckling pressure of imperfect steel cylinders under external pressure confined within a rigid cavity. Montel's predictions are shown in Figures 5, and demonstrate that this semi-empirical equation, despite its simplicity can provide very reliable estimates of the ultimate pressure and can be used for design purposes.



Figures 3. Variation of maximum pressure $p_{\max}$ steel cylinders embedded in a rigid confinement medium with respect to the slenderness parameter λ defined in equation (4); finite element results and predictions of equations (6), (10), (11).

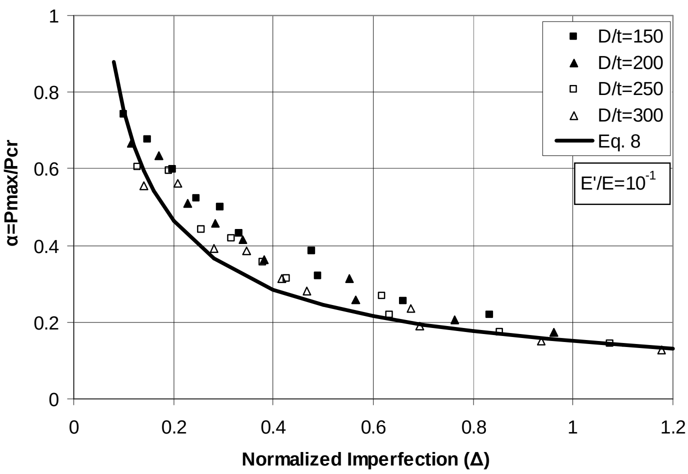
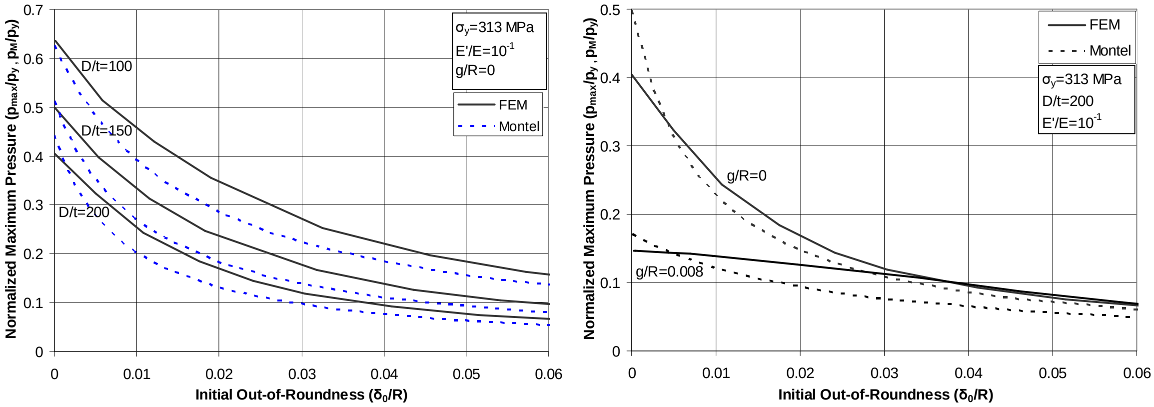


Figure 4. Imperfection sensitivity of elastic rigidly confined cylinders under external pressure; finite element results and predictions of equation (9).



Figures 5. Comparison between numerical results and analytical predictions from Montel's simplified equation (2) [4].

5. EFFECTS OF MEDIUM DEFORMABILITY

The results presented in the previous section refer exclusively to the case of cylinders enclosed within a rigid cavity, i.e. high values of the modulus of the confinement medium E'. However, quite often in buried pipeline applications the stiffness of the surrounding medium should be considered to determine the ultimate pressure of steel cylinders. Figure 6a shows the pressure versus deformation response of a steel cylinder ($\sigma_y = 313$ MPa, $D/t = 200$) with no imperfections ($\delta_0 = g = 0$), for different values of the confinement medium modulus E'. The main observation from those results is the significant reduction of the $p_{\max}$ value. Furthermore, with decreasing E' values, the response is characterized by a "plateau" on the equilibrium path about the maximum pressure.

It is possible to incorporate the effect of E'/E in the present design methodology introducing a reduction factor f, expressing the ratio of $p_{\max}$ in a deformable medium over $p_{\max,\infty}$, which is the ultimate pressure of the cylinder in a rigid confinement:

$$f = \frac{p_{\max}}{p_{\max,\infty}} \tag{14}$$

Based on the numerical results, as follows:

$$f(x) = \begin{cases} -0.05x^2 + 0.1x + 0.95 & \text{if} \quad 1 \le x \le 5 \\ 1 & \text{if} \quad x \le 1 \end{cases} \tag{15}$$

where

$$x = -\log\left(\frac{E'}{E}\right) \tag{16}$$

The comparison between numerical results and the analytical predictions is shown in Figure 6b and demonstrate that equation (15) can be used for an efficient description of the effects of medium deformability on the ultimate pressure..

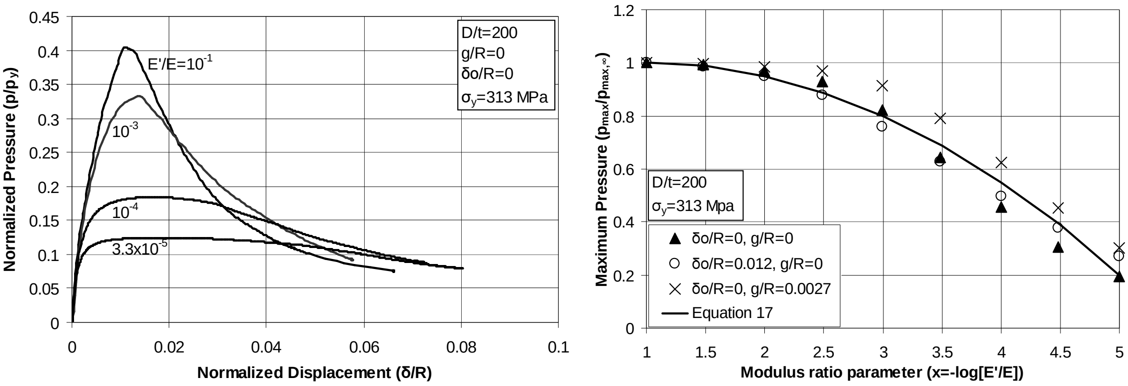


Figure 6 (a) Structural response of perfect steel cylinders for different values of confinement medium modulus (E'/E); pressure versus deformation equilibrium paths ($g/R = 0$, $\delta_0/R = 0$). (b) Comparison between numerical results and analytical prediction from equation (15) for the maximum pressure with respect to the E'/E value.

6. CONCLUSIONS

Using finite element simulation tools, buckling of cylinders confined within a deformable elastic medium is investigated. Based on the numerical results, a systematic methodology is developed for the prediction of the ultimate pressure, which also accounts for the effects of the deformability of the surrounding medium and those of initial imperfections. The methodology is compatible with the general provisions of recent European design recommendations for shell buckling [7] [8] and could be used for design purposes.

7. REFERENCES

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ΣΧΕΔΙΑΣΜΟΣ ΠΛΕΥΡΙΚΩΣ ΠΕΡΙΟΡΙΣΜΕΝΩΝ ΣΩΛΗΝΩΝ ΑΠΌ ΧΑΛΥΒΑ ΥΠΟ ΕΞΩΤΕΡΙΚΗ ΠΙΕΣΗ ΕΝΑΝΤΙ ΛΥΓΙΣΜΟΥ

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ΠΕΡΙΛΗΨΗ

Λεπτότοιχοι σωλήνες από χάλυβα, μέσα σε ένα ελαστικό παραμορφώσιμο μέσο, όταν υπόκεινται σε εξωτερική πίεση, μπορεί να πάθουν λυγισμό. Χαρακτηριστικά παραδείγματα εφαρμογών είναι οι υπόγειοι αγωγοί ύδατος, σωλήνες που χρησιμοποιούνται για την εσωτερική επένδυση παλαιών δικτύων ύδρευσης, σωλήνες/σήραγγες που μεταφέρουν υγρά και αέρια σε σταθμούς παραγωγής ρεύματος, ή σωλήνες που χρησιμοποιούνται σε πηγάδια εξόρυξης αερίου ή πετρελαίου.

Στην παρούσα εργασία, χρησιμοποιώντας μία τεχνική πεπερασμένων στοιχείων, προσομοιώνεται η δομική συμπεριφορά και ο λυγισμός λεπτότοιχων σωλήνων από χάλυβα, πλευρικώς περιορισμένων, υπό ομοιόμορφη εξωτερική πίεση. Η ανάλυση χρησιμοποιεί μη-γραμμικά πεπερασμένα στοιχεία που λαμβάνουν υπόψη γεωμετρικές μη-γραμμικότητες και ανελαστική συμπεριφορά. Ιδιαίτερη έμφαση δίνεται στην επιρροή των αρχικών ατελειών και στην παραμορφωσιμότητα του περιβάλλοντος μέσου.

Τα αποτελέσματα των πεπερασμένων στοιχείων δείχνουν μία σημαντική επιρροή στην αντοχή σε πίεση των αρχικών ατελειών και της παραμορφωσιμότητας του περιβάλλοντος μέσου. Τα αποτελέσματα χρησιμοποιούνται για την ανάπτυξη μίας απλής και αποτελεσματικής μεθοδολογίας σχεδιασμού, η οποία είναι συμβατή με τις πρόσφατες διατάξεις του Ευρωκώδικα 3 (EN 1993-1-6) και τις οδηγίες σχεδιασμού για λυγισμό κελυφών της ECCS, δημοσίευση No. 125 “European Design Recommendations for Shell Buckling”.