

# **DETERMINATION OF THE ROTATIONAL CAPACITY OF COMPACT STEEL BEAMS AT ELEVATED TEMPERATURES CONSIDERING LOCAL AND GLOBAL GEOMETRIC IMPERFECTIONS**

**Daphne Pantousa**

Civil Engineer, Msc, Doctoral Candidate

Laboratory of Structural Analysis and Design, Dept. of Civil Engineering,  
University of Thessaly,  
Volos, Greece

e-mail: [dpantousa@gmail.com](mailto:dpantousa@gmail.com)

**Euripidis Mistakidis**

Associate Professor

Laboratory of Structural Analysis and Design, Dept. of Civil Engineering,  
University of Thessaly,  
Volos, Greece

e-mail: [emistaki@uth.gr](mailto:emistaki@uth.gr)

## **1. ABSTRACT**

The objective of the paper is to propose advanced three-dimensional models based on the use of shell elements that can be used for the simulation of the structural steel behaviour under elevated temperatures. The problem is handled through thermo-mechanical analysis in the context of the finite element method. The three – dimensional numerical model that is proposed in the current study, is developed using the non-linear finite element code MSC MARC. Parametric analyses are conducted considering different amplitudes for the local and global imperfections, in order to obtain the moment – rotation curves for steel IPE beams in elevated temperatures. These curves can be used for the analysis of frame structures under fire conditions through simpler software packages, utilizing beam finite elements with concentrated nonlinear behaviour.

## **2. INTRODUCTION**

The global plastic analysis of steel structures requires that at the plastic hinge locations, the cross sections of the members which contain the plastic hinge should have rotational capacity greater than the required at this position. According to Eurocode 3, this problem is handled through the classification of the cross sections. More specifically, sufficient rotation capacity may be assumed at the plastic hinge if the cross section of the member is of Class1. In the case of the fire design of steel structures, the classification of the cross sections is performed in the same way as in room temperature, except that a factor of 0.85 is used for the calculation of  $\varepsilon$ , i.e.:

$$\varepsilon = 0.85 \left[ 235 / f_y \right]^{0.5} \quad (1)$$

where  $f_y$  is the yield strength at 20°C. As it is stated in Eurocode 3 – Part 1.2 [3], the reduction factor considers influences due to the increased temperature. This consideration could be conservative or not, since it does not take into account several factors that affect the rotational capacity of steel members under fire conditions, as the lack of the strain hardening in the stress-strain relationship after the temperature of 400°C, the effect of the initial imperfections etc. These parameters may lead to a premature occurrence of local or lateral torsional buckling in the plastic range, thus limiting the available rotational capacity.

In fact, the experimental and numerical research concerning plastic local buckling and plastic lateral torsional buckling of steel beams under fire conditions is rather limited. According to R.B. Dharma and K.H. Tan [1] the cross section classification of Eurocode 3 – Part 1.2 [3], which is based on the cross-sectional dimensions and the material yield strength, is inadequate to address the ductility of beams at elevated temperatures. This study reveals that the rotational capacity of steel I-beams is reduced at elevated temperatures. The results of the experimental program presented in [1] indicate the effect of the main parameters (web slenderness, flange slenderness, effective length) on the rotational capacity at elevated temperatures. Also, the numerical models that are developed in [2] are validated against the test results and are used for further research in order to quantify the inelastic behaviour of steel beams under fire conditions. Finally, a simple moment – rotation model is proposed that is useful for design purposes. This model takes into account all the experimental and numerical findings.

The objective of the present study is to propose an advanced three-dimensional numerical model for the evaluation of the real behaviour of steel beams under fire conditions. It must be noticed that the present three-dimensional model is based on shell finite elements and takes into account the existing initial imperfections of the steel members. Specifically, parametric analyses are conducted with respect to the amplitude of the initial imperfections, in order to obtain moment – rotation curves for steel IPE beams at elevated temperatures. Finally, moment – rotation diagrams are obtained, that take into account the reduced rotational capacity of steel beams at elevated temperatures and the effect of the initial imperfections. These diagrams can be used for the global plastic analysis of frame structures under fire conditions through more simple, commercial software packages, utilizing beam finite elements.

### 3. DESCRIPTION OF THE PROBLEM

First, a two span continuous steel I-beam under uniform loading is considered. The objective of the study is to define the rotational capacity of steel IPE beams at the possible plastic hinge locations under fire conditions. Therefore, it is important to assess the fire behaviour of the beam at the hogging moment region, i.e. at the internal supports. In order to simplify the problem, a simply supported beam is considered which is loaded at the mid-span. This beam can represent the part of a continuous beam close to the internal support, between the points where the bending moment diagram becomes zero (Figure 1a). In the case of the simply supported beam the plastic hinge will be formed at mid-span, corresponding actually to the internal support of the continuous beam.

The structural system that is considered in this study is presented in Figure 1b. In detail, the total length of the simply supported beam is equal to 2.5m. Also, web stiffeners are used at the support and at the mid-span where the load is applied. The beam is laterally restrained at the position of supports at both ends and at the mid-span. Therefore, the effective length of the beam is equal to 1.25m. The grade of the structural steel is

considered to be S275. Moreover, it is noticed that the classification of the cross section for the elevated temperatures results to Class 1.

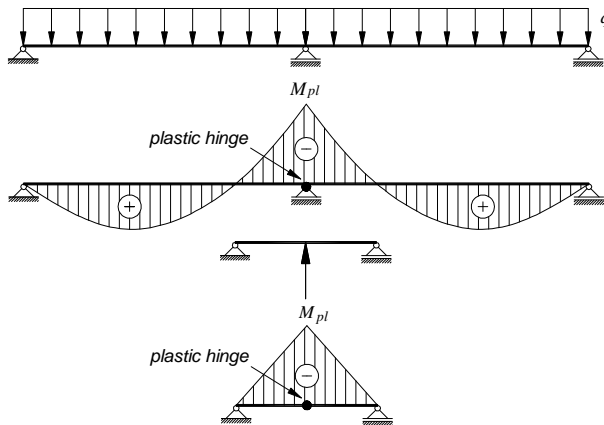


Fig. 1a: Simplification of the problem from a continuous beam to a simply supported beam.

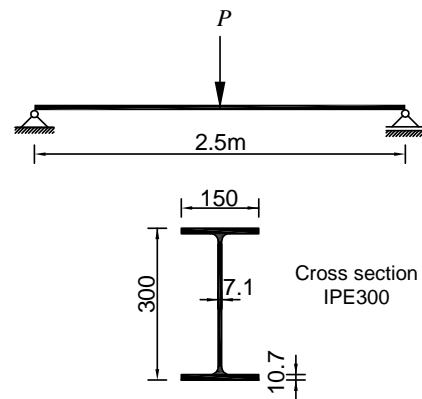


Fig. 1b: The structural system (dimensions of the cross section in mm).

It is stressed that the beam is laterally restrained only at the ends and at the mid-span and, therefore, the plastic lateral-torsional buckling is possible under certain conditions at elevated temperatures. Moreover, plastic local buckling may arise. Taking into account the latter, the problem that is handled in this study is the evaluation of the rotational capacity of a simply supported steel I-beam under fire conditions. More specifically, moment-rotation curves are obtained, for different temperatures, under the consideration that the temperature of the beam is uniform and constant. Additionally, various analyses are conducted taking into account different amplitudes of the initial geometric imperfections. It must be noticed that the beam is free to expand longitudinally, which means that the study does not take into account the effect of compressive forces. Moreover the assumption that the temperature is uniform simplifies the problem, since there is no thermal gradient at the cross section.

At this study the available rotational capacity  $r_a$  is calculated as the ratio between the inelastic rotation  $\theta_a$  and the plastic rotation  $\theta_{pl}$ , i.e.  $r_a = \theta_a / \theta_{pl}$  (see Figure 2).

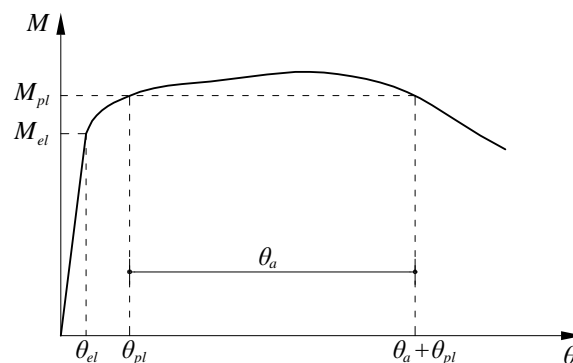


Fig. 2: Definition of the rotational capacity.

Here  $\theta_{pl}$  corresponds to the plastic moment resistance of the cross-section, defined as:

$$M_{pl,T} = f_{y,T} w_{pl} \quad (2)$$

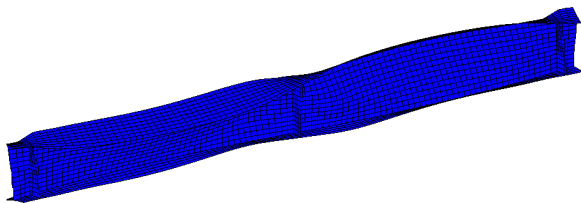
where  $f_{y,T}$  is the yield stress of steel at temperature  $T$ . The range of the rotation over which the plastic moment resistance of the cross section is retained, is called  $\theta_a$ .

## 4. THE NUMERICAL MODEL

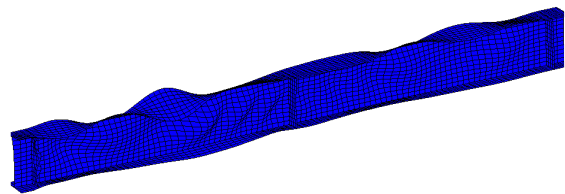
### 4.1 The numerical simulation

A detailed finite element model is proposed in order to evaluate the rotational capacity of the steel I-beam at elevated temperatures. The numerical analysis is carried out using the nonlinear finite element code MSC-MARC [4]. The three-dimensional numerical model utilizes four-node, thick-shell elements with global displacements and rotations as degrees of freedom. Bilinear interpolation is used for the coordinates, displacements and rotations and the integration through the shell thickness is performed numerically using Simpson's rule. The numerical model takes into account the nonlinear elastic-plastic stress-strain relationship of steel at elevated temperatures. The yield stress, the proportionality limit and the elastic modulus are supposed to be temperature dependent according to Eurocode 3-Part1.2 [3].

The Von Mises yield criterion is used in the numerical analysis. Additionally, the analysis takes into account the geometric non-linearity. It must be noticed that initial imperfections are incorporated in the geometry of the steel beam for a more realistic assessment of the behaviour. In general, there are many different ways to introduce initial geometric imperfections in the structural members. A simple way in the context of finite element analysis is to extract the buckling eigenmodes and introduce them as imperfections with predefined amplitude. More specifically, the normalized buckling mode is multiplied by a scale factor, leading to certain maximum amplitude and the resulting displacements are added to the initial coordinates of the structural member. For this case study two different eigenmodes are combined (see details in Figure 3). The first eigenmode is related with the local buckling along the upper flange of the beam (where compressive stresses arise under the considered loading), while the second buckling eigenmode corresponds to lateral torsional buckling.



*Fig. 3a: Eigenmode corresponding to lateral torsional buckling.*



*Fig. 3b Eigenmode corresponding to local buckling of the upper flange.*

The numerical analysis follows two different stages. At the first stage the steel beam is heated with a rate equal to  $7^{\circ}\text{C}/\text{min}$  until the desired temperature  $T$  is reached. It must be noticed that during the heating stage the temperature is supposed to be uniform along the member. At the second stage the temperature remains constant and the beam is submitted to loading at the mid-span until failure occurs.

The structural boundary conditions are described in Figure 4. At both supports the boundary conditions are applied in the middle node of the web. The vertical  $y$ -displacement and the rotation about the longitudinal- $x$  axis ( $r_x$ ) is restrained at both supports, while the  $x$ -displacement is restrained only at the left support. The out-of-plane displacements are prevented at the location of the supports and at the mid-span.

It must be noted that the numerical model has been validated against the published experimental results of [1]. The comparison of the numerical and the experimental results is presented in detail in [5].

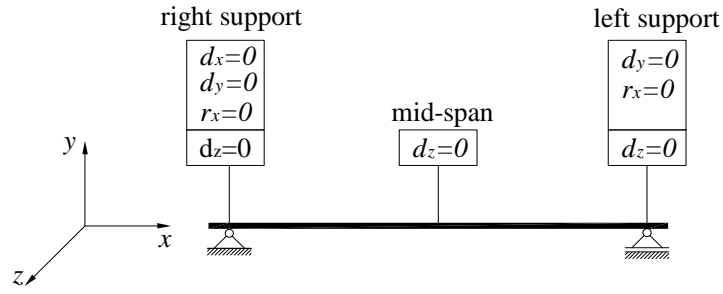


Fig. 4: The boundary conditions of the beam.

## 5. PARAMETRIC ANALYSES

Parametric analyses are conducted at various temperature levels in order to find out the effect of the initial imperfection on the available rotational capacity of the beams. In particular, the analyses are conducted for temperatures ranging between  $100^{\circ}\text{C}$  and  $800^{\circ}\text{C}$ . The amplitude of the initial imperfections is considered to be between 0.5mm and 5mm for both the local buckling and the lateral torsional buckling eigenmodes.

Four different analyses are conducted for every temperature level. Initially, the steel beam is considered to be “perfect” i.e. the initial imperfections are not taken into account. Then, parametric analyses are conducted considering amplitudes of initial imperfections equal to 0.5mm, 2mm and 5mm. The imperfection amplitudes are assumed to be the same for both the local buckling and the lateral torsional buckling eigenmodes. The dimensionless moment-rotation curves at various temperature levels are illustrated in Figure 5. The idealized elastic - perfectly plastic diagram corresponds to the design plastic moment of the cross section obtained by equation (2).

The results of the numerical analyses of the “perfect” beams indicate that the failure mode changes after the temperature of  $300^{\circ}\text{C}$ . More specifically, for  $100^{\circ}\text{C} \leq T \leq 300^{\circ}\text{C}$  the steel beam fails due to both plastic lateral torsional buckling and plastic local buckling, while for  $T > 300^{\circ}\text{C}$  the failure is only due to the appearance of the plastic local buckling. This behaviour is reflected clearly in the softening branch. It can be observed that for  $100^{\circ}\text{C} \leq T \leq 300^{\circ}\text{C}$  the softening branch is steeper compared with the corresponding curves for  $T > 300^{\circ}\text{C}$ . Additionally, the results of the numerical analyses of the “imperfect” beams indicate that the beam fails due to both plastic lateral torsional buckling and plastic local buckling for all the temperature levels.

Moreover, it can be observed that the initial imperfections do not affect significantly the maximum load bearing capacity of the beam. This result holds for all the temperature ranges. Also, it can be observed that for temperature values less than  $300^{\circ}\text{C}$  the increase of the amplitude of the initial imperfections has a minor effect to the softening branch of the diagram. On the contrary, for temperature ranges between  $400^{\circ}\text{C}$  and  $800^{\circ}\text{C}$ , as the amplitude of the imperfection increases, the softening branch becomes steeper. Also, it can be noted that for all the temperature levels the steel beam is able to reach the plastic moment resistance, since no local or lateral torsional buckling takes place in the elastic region. An exception to this can be observed for temperature equal to  $700^{\circ}\text{C}$  and amplitude of initial imperfections 5mm. For this specific case the beam is not able to reach the plastic moment capacity. Despite the fact that according to Eurocode 3 [3] for temperatures ranging between  $100^{\circ}\text{C}$  and  $300^{\circ}\text{C}$  the ultimate strength of steel is  $f_{u,T} = 1,25 f_{y,T}$ , the ultimate moment that results from the numerical analysis is  $M_{u,T} \approx 1,1 M_{pl,T}$ . This can be attributed to the fact that geometric nonlinear phenomena arise as the deflection of the beam increases.

It is clear that for temperatures ranging between  $400^{\circ}\text{C}$  and  $800^{\circ}\text{C}$ , the rotational capacity of the steel I-beam is considerably reduced. Moreover, the available rotational capacity reduces as the amplitude of the initial imperfections becomes larger. This reduction is actually a consequence of the fact that the slope of the softening branch becomes steeper as the amplitude of the imperfections increases.

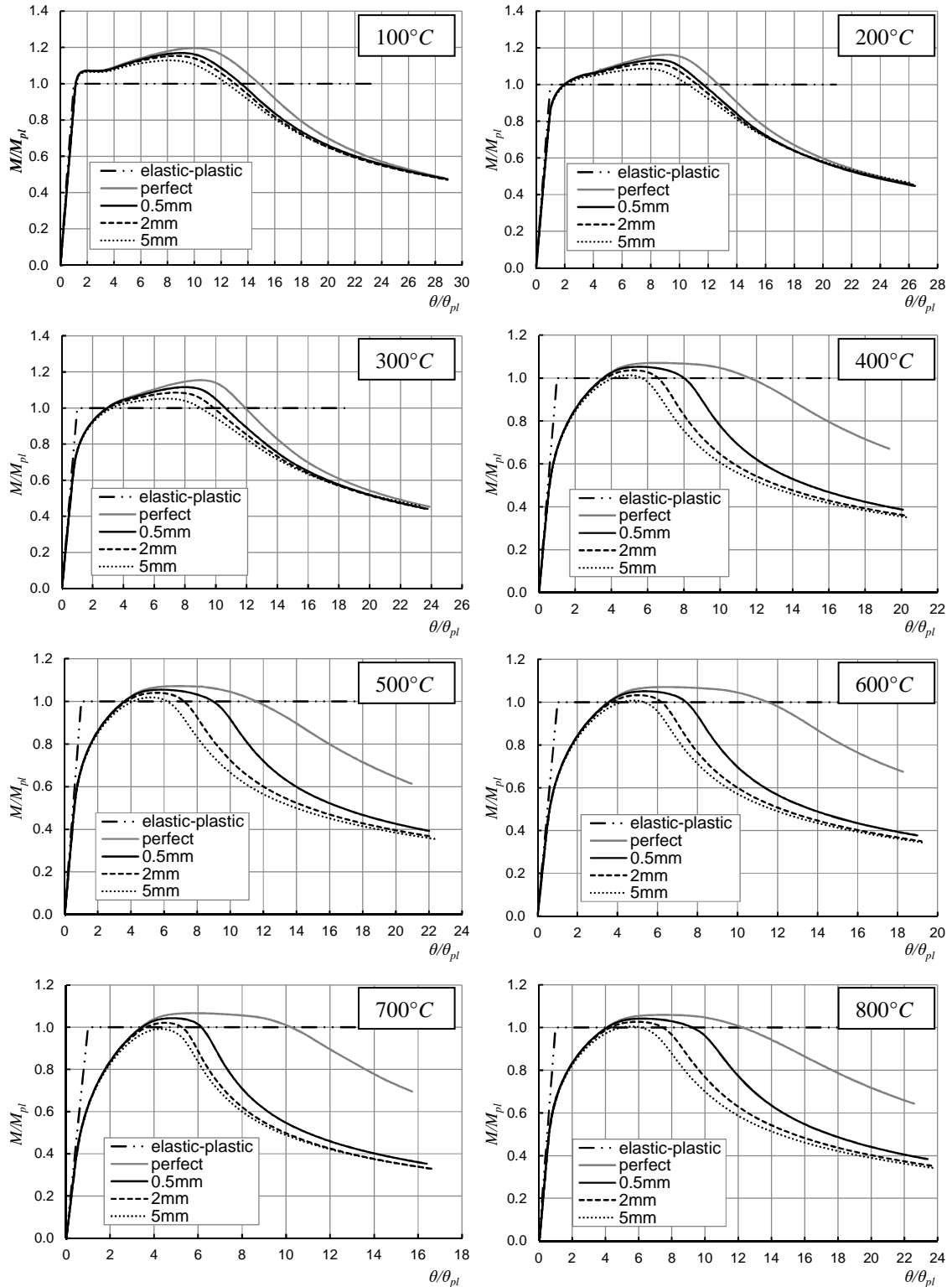


Fig. 5: Dimensionless moment rotation curves at various temperature levels.

The values of the obtained rotation capacity of the steel beam, according to the results of the parametric numerical analyses, are summarized in Table 1. It is noticed that both the initial imperfections and temperature have a significant effect on the available rotational capacity. Notice also that the rotational capacity seems to be increased for temperatures above 800°C with respect to the values obtained for lower temperatures (e.g. for 700°C).

Temperature (°C)	Amplitude of initial imperfections			
	perfect	0,5mm	2mm	5mm
100	13,69	12,22	11,91	11,18
200	10,73	9,60	7,69	8,40
300	8,96	7,59	4,49	5,74
400	8,11	4,36	2,81	1,56
500	7,98	5,32	3,45	2,06
600	7,97	3,77	2,43	1,03
700	6,83	2,64	1,56	-
800	8,21	5,13	3,10	1,07

Table 1: Available Rotational capacity of the steel beam at various temperature levels.

## 6. CONCLUSIONS

A numerical model is proposed in order to evaluate the effect of the temperature and the amplitude of initial imperfections on the rotational capacity of compact steel I-beams at elevated temperatures. The results of the parametric analyses indicate that the rotational capacity is significantly reduced for temperatures greater than 400°C. Moreover, for the same temperature range, as the amplitudes of the initial imperfections increase, a considerable reduction of the rotational capacity is noticed.

## 7. ACKNOWLEDGMENTS

The present work is financially supported by the National Research Program “HERACLITUS II”, the contribution of which is gratefully acknowledged.

## 8. REFERENCES

- [1] Dharma R.B, Tan K.H “Rotational capacity of steel I-beams under fire conditions Part I: Experimental study”, *Engineering Structures*, Vol. 29, 2007, pp. 2391-2402.
- [2] Dharma R.B, Tan K.H “Rotational capacity of steel I-beams under fire conditions Part II: Numerical simulations”, *Engineering Structures*, Vol. 29, 2007, pp. 2403-2418.
- [3] European Committee for Standardization, Eurocode 3. EN 1993-1-2 “Design of steel structures – Part 1-2. General rules – structural fire design”, 2003.
- [4] MSC Software Corporation, MSC Marc “Volume A: Theory and User Information” 2010.
- [5] Pantousa D, Mistakidis E “The effect of the geometric imperfections on the rotational capacity of steel beams at elevated temperatures”, *7<sup>th</sup> Gracm International Congress on Computational Mechanics, Athens, 2011.*

**ΕΚΤΙΜΗΣΗ ΤΗΣ ΣΤΡΟΦΙΚΗΣ ΙΚΑΝΟΤΗΤΑΣ ΧΑΛΥΒΔΙΝΩΝ ΔΟΚΩΝ ΣΕ  
ΥΨΗΛΕΣ ΘΕΡΜΟΚΡΑΣΙΕΣ ΘΕΩΡΩΝΤΑΣ ΤΗΝ ΕΠΙΡΡΟΗ ΤΩΝ ΑΡΧΙΚΩΝ  
ΓΕΩΜΕΤΡΙΚΩΝ ΑΤΕΛΕΙΩΝ**

**Δάφνη Παντούσα**

Πολιτικός Μηχανικός, Msc, Υπ. Διδάκτωρ  
Εργαστήριο Ανάλυσης και Σχεδιασμού Κατασκευών, Τμήμα Πολιτικών Μηχανικών  
Πανεπιστήμιο Θεσσαλίας  
Βόλος, Ελλάδα  
e-mail: [dpantousa@gmail.com](mailto:dpantousa@gmail.com)

**Ευριπίδης Μουστακίδης**

Αναπληρωτής Καθηγητής  
Εργαστήριο Ανάλυσης και Σχεδιασμού Κατασκευών, Τμήμα Πολιτικών Μηχανικών  
Πανεπιστήμιο Θεσσαλίας  
Βόλος, Ελλάδα  
e-mail: [emistaki@uth.gr](mailto:emistaki@uth.gr)

**ΠΕΡΙΛΗΨΗ**

Στόχος της εργασίας είναι ο υπολογισμός της στρωφικής ικανότητας μεταλλικών δοκών σε υψηλές θερμοκρασίες. Οι διατομές των μεταλλικών δομικών μελών που εξετάζονται υπάγονται στην κλάση 1, σύμφωνα με τις διατάξεις του Ευρωκώδικα 3- Μέρος 1-2 για την κατάταξη των διατομών. Το πρόβλημα αντιμετωπίζεται μέσω σύνθετης θερμομηχανικής ανάλυσης στο πλαίσιο της μεθόδου των πεπερασμένων στοιχείων. Τα τρισδιάστατα μοντέλα που παρουσιάζονται δημιουργούνται με τη χρήση του κώδικα μη-γραμμικής ανάλυσης MSC MARC. Χρησιμοποιούνται στοιχεία κελύφους έτσι ώστε να μπορούν να ληφθούν υπόψη οι αρχικές γεωμετρικές ατέλειες των δομικών μελών.

Τα προτεινόμενα αριθμητικά μοντέλα, που η αξιοπιστία τους έχει ήδη επιβεβαιωθεί σε προγενέστερη εργασία των συγγραφέων μέσω της σύγκρισης με αποτελέσματα της βιβλιογραφίας, χρησιμοποιούνται για την εξαγωγή καμπυλών ροπής-στρωφής συναρτήσει της θερμοκρασίας. Για τον σκοπό αυτό διεξάγονται αριθμητικές αναλύσεις σε διαφορετικά επίπεδα υψηλών θερμοκρασιών θεωρώντας τη θερμοκρασία της δοκού σταθερή και ομοιόμορφη. Ταυτόχρονα διαφοροποιείται το εύρος των αρχικών ατελειών με σκοπό να διερευνηθεί η επιρροή του στη στρωφική ικανότητα των δοκών στις υψηλές θερμοκρασίες.

Τα διαγράμματα που εξάγονται αποτελούν σημαντικό εργαλείο για την ανάλυση πλαισιωτών κατασκευών σε συνθήκες πυρκαγιάς με χρήση απλούστερων λογισμικών που βασίζονται σε μη-γραμμικά στοιχεία δοκού. Τα αποτελέσματα αναδεικνύουν την μείωση της στρωφικής ικανότητας των μεταλλικών δοκών με την αύξηση της θερμοκρασίας αλλά και με την αύξηση του εύρους των γεωμετρικών ατελειών.