ELASTOPLASTIC BEHAVIOR OF COLD BENT WIDE FLANGE SECTIONS

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1. ABSTRACT

Cold bending is one of the most commonly used methods for bending straight members and creating arches using hot rolled wide flange sections. This process induces plastic deformations along the length of the member so that it can develop the required curvature. Thus, a different residual stress distribution is introduced than the well-known residual stress pattern due to hot rolling. The influence of residual stresses due to hot rolling on the resistance of wide flange steel sections has been studied extensively, showing that they may have non negligible influence on the response. Limited studies have been performed using the residual stress pattern due to cold roller bending. Based on this residual stress pattern a detailed study is carried out to investigate its influence on the section resistance, focusing on their elasto-plastic behavior.

2. INTRODUCTION

Roller bending is nowadays the most common and convenient way to produce curved members. These curved members are most commonly made of hot rolled wide flange sections and are used in various structural applications such as arched roofs, atriums and bridges. Due to the manufacturing process, hot rolled wide flange sections are not stress free. They initially have an internally self-equilibrated stress distribution, the so called residual stress distribution [1]. Extensive studies have been carried out so far using this residual stress distribution showing that their influence on the resistance of such members is not negligible. This is reflected by the inclusion of buckling curves and residual stress models in construction standards such as Eurocode 3. However, after such a member is curved to its desired shape, this residual stress pattern due to hot rolling is removed and a new residual stress pattern is now induced in the sections of the curved member. In order to evaluate the elasto-plastic behavior of such sections an in-depth study of the influence of this residual stress pattern is carried out, following the methodology proposed by Sophianopoulos et al. [2] for their study on thermally induced residual stresses.

2.1. ROLLER BENT SECTIONS

2.1.1 ROLLER BENDING PROCESS

Roller bending, or cold bending as it is called, is an iterative process. A straight member passes through three rollers (Fig. 1). Plastic deformations are induced in the part of the member between the two outer rollers. As the member is rolled all over its length, plastic deformations are induced along the length of the member so that it gradually becomes an arch. In each subsequent pass, the middle roller moves vertically towards the other two, in order to increase the curvature. Because wide flange sections are prone to local buckling, there are limitations on the maximum curvature the member can assume with this process. In order to prevent local buckling, two smaller rollers are sometimes used on the tension flange, so that they prevent it from transverse bending known as “Brazier effect” or “ovalization”[3]. A detailed description of the roller bending process has been presented by Bjorhovde [4]. The elongated and shortened flanges are denoted in this paper as top and bottom flange, respectively (Fig. 1).

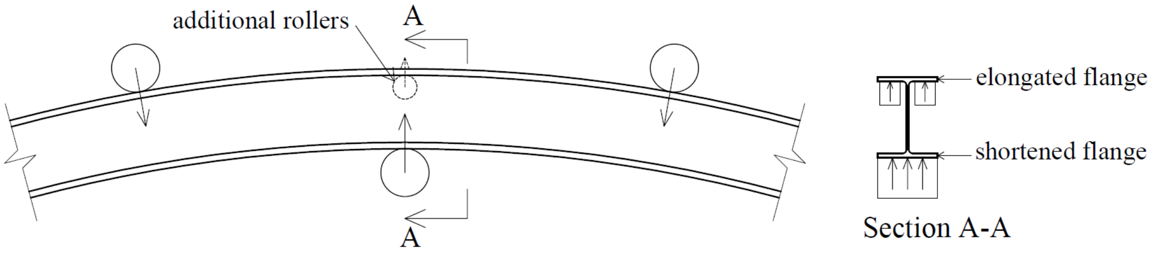

Fig. 1: Roller bending process.

2.1.2 RESIDUAL STRESSES

The residual stress distribution due to cold bending has been theoretically proposed by Timoshenko [5] using a bi-linear material law. Further analytical and experimental investigation of the residual stresses in various shapes other than wide flange sections have been performed in various studies, [6], [7], [8]. Regarding the residual stresses in wide flange sections, the experimental data are limited. Spoorenberg et al. [9] have recently carried out an experimental program which focused on the residual stresses due to cold bending on wide flange sections. Using the sectioning method for their measurements, they validated their method by comparing measurements of residual stresses in straight members with the theoretical thermal stresses and reproduced the same method for members after curving. Moreover, they compared their results with detailed finite element simulations of the roller bending process [10] by explicitly modeling the whole process. In their models they included full interaction between the machine parts, i.e. rollers, and workpiece, i.e. steel section. Based on these, they proposed a residual stress model for roller bending wide flange sections [11]. This model is generally applicable within a range of bending radii of $10 \leq R/h \leq 40$ and is linearly related to the magnitude of the original yield stress for S235 and S355 steel sections. In this model the residual stress gradients over the web thickness and the flange thickness are ignored. For the top flange, a linear stress gradient of 0,20f_y tensile stress at the flange tips to -0,20f_y compressive stress at the flange center is proposed. For the bottom flange a bi-linear stress pattern is suggested, with a compressive stress of 0,35f_y at the flange tips and a maximum of 0,70f_y at the flange center. For the web two triangular stress blocks are suggested, namely a tensile and a compressive stress block with the peaks of the triangles at distances of 0,25h_0 and 0,75h_0 respectively from the web-to-flange junction of the top flange. Based on equilibrium conditions the value of the peaks for the tensile stress block is:

$$\sigma_{wrt} = \frac{7bt_f}{30h_0t_w} f_y \tag{1}$$

The corresponding compressive stress block peak is given by:

$$\sigma_{wrc} = -\frac{14bt_f}{30h_0t_w} f_y \tag{2}$$

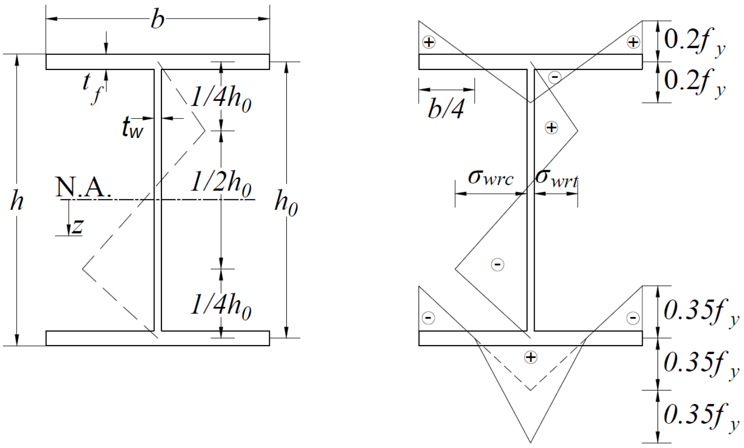

Fig. 2: Proposed residual stress model for roller bent I-sections by Spoorenberg et al.[11].

2.1.3 INFLUENCE ON RESISTANCE

Studies on the influence of residual stresses of roller bent steel arches which have been performed by various researchers [12], [13], were based on a residual stress pattern of straight members. However, recent studies [9], [10], [11] showed that this is not the actual residual stress distribution of such members. As data on the actual residual stress pattern are few and quite recent, there is still a lack of studies on their influence on section and member resistance.

3. ELASTOPLASTIC BEHAVIOR OF ROLLER BENT WIDE FLANGE SECTIONS

The residual stress distribution proposed by Spoorenberg et al. [11] is well defined and is currently the most realistic residual stress model for roller bent wide flange sections. According to this model (Fig. 2) there is a non-symmetrical stress distribution through the height of the section with significant magnitude on the bottom flange and around the peaks of the triangles on the web. The residual stress magnitude in the flanges depends on the yield stress f_y. Residual stress magnitude in the web depends also on the geometric properties of the section. As this stress model is limited to circular arches curved about the strong axis, the study focuses on this case only. The residual stress pattern of Spoorenberg et al. [11] is used as an initial stress state in wide flange sections in order to investigate their elasto-plastic behavior. Based on the methodology of Sophianopoulos et al. [2] an in-depth study of the influence of the residual stresses is conducted. The study is decomposed into two parts: first the determination of the elastic domain and then that of the plastic domain. The material is assumed to be elastic-perfectly plastic, neglecting hardening. The plus (+) sign denotes tensile axial forces and moments that tend to open the arch , while the opposite holds for the negative sign (-).

3.1. DETERMINATION OF THE ELASTIC DOMAIN

The normal stress at any position over the height of the section should always be less or equal to the yield stress f_y, for any combination of normal force N and bending moment M_N, including the residual stresses (Fig. 2). This implies that for various combinations of these loads, the stress in the section will first reach the value of f_y in one of the peaks of the residual stress model. Which peak will first reach the yield stress f_y depends on the magnitude and direction of normal force and moment. The general expression is:

$$\frac{N}{A}+\frac{M_N}{I}z+\sigma_{rs}\leq f_y \tag{3}$$

z is the distance from the neutral axis (N.A.) of the section, and σ_{rs} is the residual stress depending on the position over the height of the section. By satisfying expression (3) at every position over the height of the section, the elastic domain is defined.

3.2. DETERMINATION OF THE PLASTIC DOMAIN

For the plastic domain, it is supposed that the central part of the section undertakes the normal forces and the external parts the bending moments, in order to maximize the section's capacity. The limit between these parts is denoted as y_N and it is evaluated from the magnitude of normal forces and the geometrical characteristics of the section. The normal force in the section is:

$$N = 2t_w y_N f_y - \int_S \sigma_{rs} dz \tag{4}$$

where $S = y_N t_w$ .The corresponding moment is:

$$M_N = M_p - M_{py_N} - \int_F \sigma_{rs} z dF \tag{5}$$

where $F = A - y_N t_w$, M_p is the total plastic moment resistance of the section and M_{py_N} is the moment within the range of y_N, in other words the part of the section which undertakes the normal forces. Repeating the calculations shown above for all values of y_N, namely from zero to the whole height of the section, including the residual stresses, the plastic domain of the section can be defined. The values of the bending moment and normal forces are normalized with the total plastic capacity of the section and the total axial capacity of the section.

3.3. PARAMETRIC STUDY

Based on the residual stress model of Spoorenberg et al. [11] and on the proposed methodology for obtaining the elastic and plastic domain, a parametric study is carried out. The yield stress has no influence on the elasto-plastic behavior of the section, as it is linearly related to the residual stress model. On the other hand, the geometric characteristics of the section influence the results. This is reflected in expressions (1) and (2) for the residual stresses on the web. By changing the geometric characteristics of the section and specifically the non-dimensional parameter bt_f/h_0t_w the magnitude of the residual stresses on the web changes, whereas they remain constant on the flanges, independent from any geometric characteristics. This implies that the influence of residual stresses is different for different sections. The higher this non-dimensional parameter is, the higher the residual stresses will result to be on the web. Consequently their influence on the limits of both the elastic and plastic domain depends on this non-dimensional parameter.

Since the residual stress model used in this paper is especially developed for wide flange sections, the study is focused on the most common sections of the European steel industry, IPE, HEA, HEB. Among these sections this non-dimensional parameter varies considerably. In IPE sections this parameter is smaller, varying from 0.5595 (IPE600) to 0.8415 (IPE80). For HEA/HEB sections this parameter is, in general, larger, varying between 0.5877(HEA1000) and 1.8763 (HEB260).

Using the procedure described in sections 3.1 and 3.2, interaction diagrams are developed for both the determination of the elastic and the plastic domain for the cases that this parameter obtains maximum and minimum values. Fig. 3 to 6 represent the interaction diagrams for the above sections.

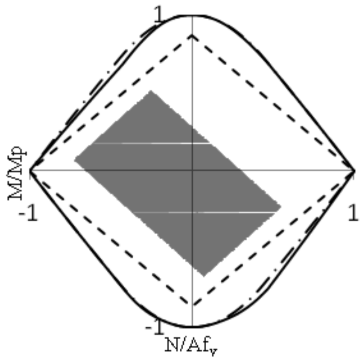

Fig. 3: Interaction diagrams for IPE600.

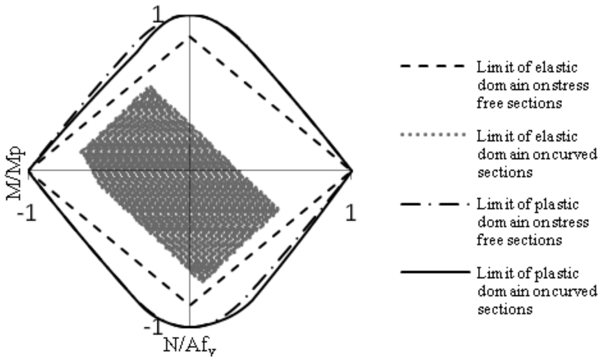

Fig. 4: Interaction diagrams for IPE80.

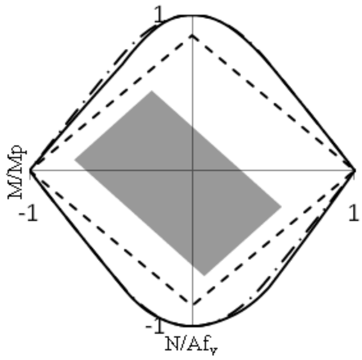

Fig. 5: Interaction diagrams for HE1000A.

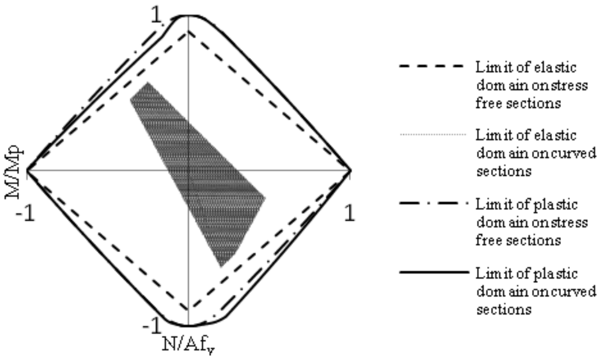

Fig. 6: Interaction diagrams for HE260B.

4. DISCUSSION AND CONCLUSIONS

4.1. RESIDUAL STRESS MODEL

The proposed residual stress model by Spoorenberg et al.[11] is currently the only one developed specifically for roller bent wide flange sections. Even though it is based on detailed experimental and state-of-the-art computational study, it should be verified by further investigation. It is independent of the radius of curvature of the bent member. It is linearly related to the yield stress of the material law, which is consistent with the theoretical model of Timoshenko [5]. However, the magnitude and variation of stresses between the two models are different, especially on the flanges. This might result from the fact that the theoretical model [5] assumes a bilinear material law without strain hardening. while strain hardening is included in the experimental [9] and computational [10] study of Spoorenberg et al. Additionally, the theoretical model assumes no stress variation over the width of the flanges, which may not be realistic. Considering the fact that during bending the section undergoes large strains, the assumption that plane sections remain plane might not be valid. Also, the interaction of the rollers on the roller bending machine and the member might also alter the theoretical residual stress model. Further analytical study is required in order to better study these effects.

4.2. ELASTO-PLASTIC BEHAVIOR OF ROLLER BENT SECTIONS

The residual stresses affect both the elastic and plastic domain of roller bent wide flange sections. Since the residual stress model used in this study is not symmetric, its influence depends on the type of loading (positive or negative moments and tensile or compressive normal forces). The elastic domain in curved sections is significantly reduced due to the presence of residual stresses. The plastic domain limit is affected a lot less, but in some cases it is reduced by approximately 10%, so that an additional safety margin of this magnitude seems reasonable for such sections. The non-smooth limits of the plastic domain at some locations on the interaction diagram are due to the different residual stress variation on the web and the flanges of the section. The influence of the residual stresses increase with the increase of the non-dimensional parameter bt_f/h_0t_w which determines the residual stresses on the web. In general HEA/HEB sections are affected more by the presence of residual stresses than IPE sections, since this parameter is higher in such section types. It is pointed out that such curved members are also prone to stability issues, which are also affected by the residual stress distribution. This effect is investigated in an on-going study.

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ΕΛΑΣΤΟΠΛΑΣΤΙΚΗ ΣΥΜΠΕΡΙΦΟΡΑ ΕΝ ΨΥΧΡΩ ΚΑΜΠΥΛΩΜΕΝΩΝ ΔΙΑΤΟΜΩΝ ΔΙΠΛΟΥ ΤΑΥ

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ΠΕΡΙΛΗΨΗ

Η μέθοδος της εν ψυχρώ καμπύλωσης είναι σήμερα η πιο συνηθισμένη διαδικασία για την παραγωγή τοξοτών μελών από ευθύγραμμα μέλη με τη χρήση πρότυπων διατομών. Κατά τη διαδικασία της καμπύλωσης οι διατομές του μέλους πλαστικοποιούνται σε όλο τους το μήκος, ώστε αυτό να αποκτήσει την επιθυμητή ακτίνα καμπυλότητας. Αποτέλεσμα αυτής της διαδικασίας είναι να αναιρείται η όποια προϋπάρχουσα κατανομή παραμενουσών τάσεων και να προκύπτει μια νέα. Η επιρροή των παραμενουσών τάσεων λόγω της εν θερμώ έλασης έχει εξεταστεί εκτεταμένα και προκύπτει ότι σε αρκετές περιπτώσεις αυτή πρέπει να λαμβάνεται υπόψη. Αντίθετα η διερεύνηση της επιρροής των παραμενουσών τάσεων σε διατομές μελών που έχουν καμπυλωθεί εν ψυχρώ είναι περιορισμένη. Σε αυτή την εργασία εξετάζεται η επιρροή που έχει αυτή η κατανομή παραμενουσών τάσεων σε διατομές διπλού ταυ. Προκύπτει πολύ έντονη επιρροή στο όριο ελαστικής συμπεριφοράς, και αισθητά μικρότερη, αλλά σε κάποιες περιπτώσεις όχι αμελητέα, επιρροή στην πλαστική αντίσταση της διατομής.