ROBUSTNESS OF STEEL AND COMPOSITE BUILDING STRUCTURES

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ABSTRACT

Recent events such as natural catastrophes or terrorism attacks have highlighted the necessity to ensure the structural integrity of buildings under an exceptional event. According to the Eurocodes and some different other national design codes, the structural integrity of civil engineering structures should be ensured through appropriate measures but, in most cases, no precise practical guidelines on how to achieve this goal are provided. At Liège University, the robustness of building frames is investigated with the final objective to propose design requirements to mitigate the risk of progressive collapse considering the conventional scenario “loss of a column” further to an unspecified event. In particular, a complete analytical procedure has been developed for the verification of the robustness of steel or composite plane frames. For sake of simplicity, these first works have been based on the assumption that the dynamic effects linked to the column loss were limited and could therefore be neglected. More recently, complementary works have been carried out with the objective to address the dynamic effects. Besides that, the extension of the static procedure to actual 3D frames is under investigation in Liège. The present paper gives a global overview of the ongoing researches in the field of robustness at Liège University and, in particular, the global strategy aiming at deriving design requirements is detailed.

1 ADOPTED STRATEGY

The studies performed at Liège University in the field of “robustness of structures” are mainly dedicated to the exceptional scenario “loss of a column” in a steel or steel-concrete composite building structure. The main objective is to derive guidelines for an appropriate design of the structure for the considered scenario. To achieve this goal, simplified analytical procedures are developed to predict the response of the structure further to a column loss; as an outcome, the way on how each structural parameter influences the structural behaviour may be described. The present section describes the global research strategy adopted by the authors.
The loss of a column can be associated to different types of exceptional events: explosion, impact of a vehicle, fire… Under many of these exceptional actions, dynamic effects may play an important role. However, it is first assumed that the column loss does not induce dynamic effects; so, the investigations of the structural response may be founded on static approaches. A building structure losing a column can be divided in two main parts, as illustrated in Fig. 1:

- the directly affected part which represents the part of the building which is directly affected by the column loss, i.e. the beams, the columns and the beam-to-column joints which are just above the failing column and;
- the indirectly affected part which includes the rest of the structure. The indirectly affected part is affected by the loads developing within the directly affected part; but obviously, these forces are themselves influenced by the response of the indirectly affected part.

If a cut is realised in the structure at the top of the failing column (see Fig. 1), different internal forces in the vertical direction are identified: (i) the shear loads $V_1$ and $V_2$ at the extremities closed to the failing column, (ii) the axial load $N_{up}$ in the column just above the failing column and (iii) the axial load $N_{lo}$ in the failing column. The objective is to predict the evolution of the vertical displacement of point “A” $\Delta_A$ according to $N_{lo}$, with due account to the eventual membrane forces developing in the structure, in order to know the requested ductility of the different structural members and to check the resistance of the indirectly affected part loaded by additional loads coming from the directly affected part.

In Fig. 2, the curve representing the static evolution of the vertical displacement $\Delta_A$ according to the normal load $N_{lo}$ in the failing column (see Fig. 1) is illustrated:

- From point (1) to (2) (Phase 1), the design loads are progressively applied, i.e the “conventional” loading is applied to the structure; so, $N_{lo}$ progressively decreases ($N_{lo}$ becomes negative as the column “AB” is subjected to compression) while $\Delta_A$ remains approximately equal to 0 during this phase. It is assumed that no yielding appears in the investigated frame during this phase, i.e. the frame remains fully elastic.

- From point (2) to (5), the column is progressively removed. Indeed, from point (2), the compression in column “AB” $N_{lo}$ decreases until it reaches a value equal to 0 at point (5) where the column is considered as fully destroyed. So, in this zone, the absolute value of $N_{lo}$ progressively decreases while the value of $\Delta_A$ increases. This part of the graph is divided in two phases as represented in Fig. 2:
From point (2) to (4) (Phase 2): during this phase, the directly affected part passes from a fully elastic behaviour (from point (2) to (3)) to a global plastic mechanism. At point (3), the first plastic hinges appear in the directly affected part.

From point (4) to (5) (Phase 3): during this phase, high deformations of the directly affected part are observed and second order effects play an important role. In particular, significant catenary actions develop in the bottom beams of the directly affected part.

It is only possible to reach point (5) if:

- resistance of the directly affected part is appropriate;
- the loads which are reported from the directly affected part to the indirectly affected part do not induce the collapse of elements in the latter (for instance, buckling of columns or development of a global plastic mechanism in the indirectly affected part);
- the different structural elements have a sufficient ductility to reach the vertical displacement corresponding to point (5).

This global approach was first developed for steel and composite structures but may be applied to other typologies of structures, as given in Table 1.

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Table 1. Steps to be crossed to derive design recommendations
In a first step, simplified analytical methods were developed to predict the response of 2D steel and composite frames further to the loss of a column with no dynamic effects; the latter are summarised in section 2. Then, based on this first step, studies were initiated to take the 3D structural response and the dynamic effects into account; these two aspects are respectively addressed in Section 3 and Section 4. The final objective is to progressively complete Table 1 with “D” indexes, what means that design recommendations would have been derived for most typologies of structures, with a similar global approach.

2 STATIC BEHAVIOUR OF 2D FRAMES FURTHER TO A COLUMN LOSS

Luu, in his PhD Thesis [2] studied the static response of 2D frames further to a column loss during Phase 1 and 2 (Fig. 2), while the PhD Thesis of Demonceau [1] concentrates on Phase 3 in which catenary effects develop. The adopted strategy to study Phase 3 is presented in Fig. 3:
- Step 1: an experimental test is carried out in Liège on a substructure with the aim to simulate the loss of a column in a composite building frame;
- Step 2: analytical and numerical FEM tools are validated through comparisons with the experimental results;
- Step 3: parametric studies based on the use of the models validated at step 2 are carried out; the objective is to identify the parameters influencing the frame response during Phase 3;
- Step 4: a simplified analytical method is developed with due account of the parameters identified at step 3 and validated through comparisons with the experimental test results of step1.

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<th>STEP 1: experimental test on a substructure simulating the loss of a column</th>
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<td>STEP 2: validation of the numerical and analytical tools</td>
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<td>STEP 3: parametric numerical studies</td>
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<td>STEP 4: development of simplified analytical methods</td>
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Derivation of design guidelines for practitioners

Fig. 3. Strategy followed to investigate Phase 3

In the present paper, part of the research works performed within steps 1 (Section 2.1) and 4 (Section 2.2) are reflected. More information is available in [1] and [3].

2.1 Experimental test on a substructure simulating the loss of a column

A test on a composite substructure has been performed to simulate the loss of a column. The main objective of the test was to observe the development of catenary actions within a frame and the effect of these actions on the behaviour of the semi-rigid and partial-strength composite beam-to-column joints. Indeed these joints are initially designed and loaded in bending, but have progressively to support tensile loads as a result of the development of membrane tying forces in the beams.
To define the substructure properties, an “actual” composite building was first designed [1] according to Eurocode 4, so under “normal” loading conditions. As it was not possible to test a full 2-D actual composite frame within the project, a substructure was extracted from the actual frame [1]; it was chosen so as to respect the dimensions of the testing floor in the laboratory but also to exhibit a similar behaviour than the one in the actual frame. The tested substructure is presented in Fig. 4. As illustrated, horizontal jacks were placed at each end of the specimen so as to simulate the lateral restraints brought by the indirectly affected part of actual building when catenary actions develop.

A specific loading history was followed during the test. First, the vertical jack at the middle was locked and permanent loads were applied on the concrete slab with steel plates and concrete blocks (“normal” loading situation). Then, the vertical jack was unlocked and large displacements develop progressively at point A (Fig. 4) until the force in the jack vanished (free spanning of 8 m). Finally, a downward vertical displacement was imposed to the system above the impacted column and was then progressively increased until collapse. The “vertical load vs. vertical displacement at point A” curve is reported in Fig. 5.

![Tested substructure](image)

**Fig. 4. Tested substructure**

![Vertical load at the jack vs. vertical displacement at point A](image)

**Fig. 5. “Vertical load at the jack vs. vertical displacement at point A” curve**

The first part of the test is represented by the segment “OA” of the curve presented in Fig. 5 and which represents the evolution of the vertical load acting on the beams at the middle of the substructure according to the vertical displacement under the “impacted” column. The vertical reaction in the lower column stub, before its removal, is equal to -33.5 kN (value of the load at point “O”). From Fig. 5, it can be seen that the structure remains globally elastic when “A” is reached.
Then, as previously explained, a increasing vertical displacement is progressively imposed until failure. During this stage, two “unloading-reloading” sequences are followed as illustrated in Fig. 5.

From point “A” to “B” in Fig. 5, the substructure yields progressively to finally form a beam plastic mechanism at point “B” (development of plastic hinges in the joints). At that moment, the cracks in the concrete slab at the external composite joints are pronounced and yielding of some steel components of the joints is observed (column web and beam flange in compression). Also, for the internal composite joint, a detachment of the end-plate and of the column flange is observed.

From point “B” to “C”, a plateau develops, what means that the vertical displacements increase with a constant vertical load (equal to 30 kN). All along the plateau, the concrete cracks in the vicinity of the external composite joints continue to extend and yielding spreads further in the steel joint components. Besides that, the concrete in compression close to the internal composite joint crushes.

The horizontal jacks begin to be significantly activated at point “C” in Fig. 5; at this point, membrane forces start to develop as confirmed by the shape of the global displacement curve (part “CD”). At point “D”, the longitudinal rebars in the external composite joints suddenly fail; at that moment, the external joints work as steel ones. Yielding also affects the different components of the internal and external joints. At point “D”, a loss of stiffness related to the failure of the rebars is observed; indeed, when these rebars fail, both flexural and tensile stiffness of the external joints decrease; but this not prevent the further development of catenary actions.

Indeed, it can be observed that the failure of the rebars does not lead to the failure of the substructure; after point “D”, the vertical load at the vertical jacks still increases with the imposed displacement (part “DE” of the curve in Figure 24).

This is possible as long as the steel connection is able to support, alone, the membrane forces developed in the system. In addition, associated to the loss of the rebars, the vertical displacements are increasing with a low variation of the vertical loads. These additional vertical displacements induce an increase of the membrane forces. So, the steel connection working alone has at the end to be sufficiently resistant to support these additional membrane forces and sufficiently ductile to support the additional rotations associated to the vertical displacement. The capacity of the steel connections, working alone, to support significant membrane forces has been confirmed by tests on joints in isolation performed at Stuttgart University [4].

2.2 Prediction of the frame response during Phase 3

In [1], it was shown, through numerical investigations, that it is possible to extract a simplified substructure (see Fig. 6) composed of the beams and the joints just above the lost column and likely represent accurately the actual global response of full frame during Phase 3. Accordingly, a simplified analytical method based on a rigid-plastic analysis has been developed to predict the response of the so-defined substructure. Also, as the deformations of the substructure are significant and influence its response, a second-order analysis has been conducted.
The parameters taken into account in this process are illustrated in Fig. 6:

- \( p \) is the (constant) uniformly distributed load applied on the storey modelled by the simplified substructure and the concentrated load
- \( Q \) is a concentrated load simulating the progressive loss of resistance of the column (= \( N_{lo} - N_{up} \); see Fig. 1);
- \( L \) is the total initial length of the substructure;
- \( \Delta Q \) is the vertical displacement at the concentrated load application point;
- \( \delta_k \) is the deformation of the horizontal spring simulating the lateral restraint provided by the indirectly affected part;
- \( \delta_{N1} \) and \( \delta_{N2} \) are the plastic elongations at each plastic hinge;
- \( \theta \) is the rotation in the plastic hinges at the beam extremities.

In addition, the axial and bending resistances at the plastic hinges \( N_{Rd1} \) and \( M_{Rd1} \) for plastic hinges 1 and 4 and \( N_{Rd2} \) and \( M_{Rd2} \) for plastic hinges 2 and 3 have also to be taken into account (it is assumed that the two plastic hinges 1 and 4 and the two plastic hinges 2 and 3 (see Fig. 6) have respectively the same resistance curve for M-N interaction).

So as to be able to predict the response of the simplified substructure, the stiffness \( K \) and the resistance \( F_{Rd} \) of the lateral restraint have to be known; these parameters depend of the properties of the indirectly affected part (see Fig. 1). In [2] and [5], analytical procedures have been defined to predict these characteristics.

In [1], Demonceau proposes an analytical expression for the \( Q-\Delta Q \) curve characterising the response of the simplified substructure. As a validation, the results obtained with the latter have been compared to the results of the experimental test performed on the substructure (see previous section). In Fig. 7, it is seen that a very good agreement is obtained between the analytical prediction and the experimental measurements. More details about the developed method are available in [1].

3 STATIC BEHAVIOUR OF 3D STRUCTURES FURTHER TO A COLUMN LOSS

In [6], the behaviour of 3D structures made of steel beams and columns has been investigated. Two different structures have been considered, with same dimensions and constitutive elements (see Fig. 8); they just differ by the joint properties at the extremities of the secondary beams: pinned joints in Structure 1 and fully rigid joints in Structure 2.

For both cases, the column which is considered to be lost is the central one, as illustrated in Fig. 8 (column “BX”).
For each structure, a simplified substructure (see Fig. 9) has been defined and extracted from the full 3D structure with the objective to check the possibility of this substructure to simulate with a sufficient accuracy the behaviour of the actual structure when significant membrane forces develop. The procedure followed for the definition of the substructure is the same as the one used for 2D frames (see Section 2 and [1]). This substructure is made of (i) four beams (two primary beams and two secondary beams) connected at the top of the failing column and of (ii) the joints at the extremities of these beams.

The influence of the rest of the structure (i.e. the part which is not directly affected by the column loss) is reflected by horizontal springs at the extremities of the so-defined substructure (see Fig. 6), with appropriate stiffness ($K_x$ and $K_z$).

In Fig. 10, a comparison between the predictions obtained (i) through a numerical simulation of the global 3D structure losing a column and (ii) through a numerical simulation of the so-defined substructure is given for the two considered structures. The graphs given in Fig. 10 represent the evolution of the axial load $N_{lo}$ in the failing column according to the vertical displacement at the top of this column. As the objective with the substructure is to predict the behaviour of the structure when significant membrane forces develop in the system, the predictions can only be compared from point A (see Fig. 10), i.e. when a plastic mechanism is formed in the structure and significant vertical displacements are reached. In Fig. 10, it can be observed that a very good agreement is obtained for Structure 1 while it is not the case for Structure 2.

This observation can be explained as follows. The loss of the column is reflected in the substructure modelling through the application of a concentrated load $Q$ (see Fig. 9). In practice, this load $Q$ is equal to the difference between $N_{lo}$ and $N_{up}$ (see Fig. 1). For some structures, it was demonstrated through a parametrical study [2] that, when significant membrane forces are developing in the directly affected beams, the value of $N_{up}$ can be assumed as a constant. Accordingly, the variation of $Q$ vs. the deformation of the substructure reflects the variation of $N_{lo}$ in the global structure. It is the reason why, for some 2D structure, it is possible to reflect the actual behaviour of the 2D frame with the substructure. For Structure 1, $N_{up}$ remains approximately constant after the formation of the plastic mechanism and thus the substructure approach is valid. But for Structure 2, $N_{up}$ in not remaining constant and, as a result, the variation of $Q$ according to the vertical displacement in the substructure modelling does not reflect the actual evolution of $N_{lo}$ in the 3D structure. The fact that $N_{up}$ is no more constant when significant membrane forces are developing is linked to the fact that a redistribution of forces takes place between the storeys located above the lost column; this aspect, which has to be explicitly considered in the model, has not been analytically characterised yet but is currently investigated. If the variation of the normal force in the column just above the failing one is introduced in the substructure model, it may be seen that the results are in good agreement with those obtained from the study of the actual full 3D structure.

It is also demonstrated in [6] that the analytical method initially developed for 2D frames [1] and able to predict the response of the “2D” substructure can be easily adapted to predict the response of the “3D” substructure defined in Fig. 9. Accordingly, when a method will be available to predict the influence of the restraint provided by the upper storeys on the normal load in the column just above the failing one, it will be possible to
predict analytically the behaviour of the global 3D structure through the substructure modelling.

Fig. 8. Investigated 3D structure

Fig. 9. Substructure extracted from the 3D structure

Fig. 10. Comparisons between the results obtained through numerical simulations (i) of the 3D structure and (ii) of the substructure

4 DYNAMIC BEHAVIOUR OF 2D FRAMES FURTHER TO A COLUMN LOSS

In [7] (see also [8]), the dynamic behaviour of 2D steel frames further to a column loss has been studied. In particular, a simplified model has been developed to predict the dynamic behaviour of the substructure defined in Section 2. Some details of the conducted investigations are given here after.
4.1 Description of the considered substructure and loading

The dynamic behaviour of a simplified substructure such as described above was investigated under the following assumptions: steel structures are considered, the material behavioural laws are supposed not to be affected by strain rate effects and a quasi-static elastic-perfectly plastic material law is assumed (infinite ductility), the stiffness $K$ of the lateral spring remains constant and, finally, the beam-to-column joints are perfectly rigid and fully resistant.

A uniformly distributed load $p$ is applied on the double-beam. Initially, the central support is present and sustains a force $N_0$ ($N_0 = p.l_0$, $l_0$ being the initial length of each beam). Then, the latter is progressively removed, which is simulated by the application of a force $p$ equal and opposite to $N_0$ in the middle of the system. The complete loss of the support takes a time $t_r$ and a linear decrease of the force it sustains is assumed. In static conditions, it had been shown in [1] that the uniformly distributed load $p$ could be neglected as far as the behaviour in phase 3 was investigated, i.e. for $p$ greater than $P_{pl}$, which is the force corresponding the plastic plateau in the static curve (development of a beam mechanism). The validity of this assertion for dynamic situations was studied [7]. Many numerical dynamic tests were made on a substructure in order to compare the maximum displacement obtained in the two loading situations (Fig. 11) for the same loading parameters $P$ and $t_r$ (Fig. 12). It was observed that the difference is limited provided the force $P$ is great enough (above the static plastic plateau). That is the reason why the behaviour of the substructure under the simplified loading situation was mainly investigated. Moreover, it was shown that the introduction of damping in the system does not induce a significant decrease of the maximum displacement [7]. As a consequence, undamped systems were considered; this constitutes a conservative approach.

![Fig. 11. Considered system with realistic and simplified loadings](image)

4.2 Influence of different parameters on the dynamic behaviour

To investigate the dynamic response of the substructure, a simplified loading is considered, consisting of a single concentrated load applied in the middle of the system. Obviously, as well as under static loading, the level of the load $P$ and the geometrical and mechanical characteristics of the structure have an influence on its behaviour. In case of dynamic loadings, the application rate of $P$, characterised by the rise time $t_r$ (Fig. 12), is also important. Besides, mass and damping properties are essential factors on which the dynamic response depends.
As mentioned above, the studied systems are undamped ones. As far as mass influence is concerned, a change in mass has first the effect of modifying the principal natural period of the system. Numerical tests proved that the dynamic response of a given structure is actually governed by two parameters: $P$ and $t_r/T$, where $T$ is the period of the principal eigenmode in the elastic domain [7]. Thus, if the mass of the system is modified but the rise time of the load is adapted so that the ratio $t_r/T$ is kept constant, then the maximum displacement remains unchanged. Furthermore, the time evolution of the displacement remains the same provided it is expressed as a function of a non-dimensional time $t/T$ (or $t/t_r$).

In [7], the behaviour of the substructure according to the loading parameters $P$ and $t_r$ (or $t_r/T$) was investigated through numerical dynamic analyses. All the results presented below are related to the following particular substructure:

- beams: $l_0 = 6.5\,\text{m}$, IPE 450, S235, $m = 3000\,\text{kg/m}$ ($T = 0.31\,\text{sec}$);
- spring: stiffness $K = 10000\,\text{kN/m}$ and very high resistance $F_{ld}$.

Performing dynamic analyses for different loading conditions ($P$, $t_r$) and registering the maximum displacement $u_{\text{max}}$ obtained for each one, curves giving $u_{\text{max}}$ as a function of the applied force $P$ were established, for different values of $t_r$ (constant along one curve). These curves are drawn in Fig. 13; only dynamic loadings leading to $u_{\text{max}}$ smaller than the displacement corresponding to the complete yielding of the beams in tension are considered. On this graph, the upper curve is the static one, while the lower curve is the so-called pseudo-static one, which gives the maximum displacement reached if $P$ is applied instantaneously ($t_r = 0$). Such a curve can easily be established provided only the nonlinear static curve is known, following a procedure developed at London Imperial College [4]. Obviously, the maximum displacement corresponding to a force $P$ will always be situated between the static displacement ($t_r \to \infty$) and the displacement caused by the sudden application of the load ($t_r = 0$). As a consequence, every $(P,u_{\text{max}})$ curve will lie between the static and the pseudo-static ones all along, for any value of $t_r$. As a general rule, for a given value of $(P,u_{\text{max}})$ tends to decrease when $t_r$ increases.

![Fig. 13. Maximum dynamic displacement according to the value of the load and its rise time](image)

Different types of behaviour can already be highlighted from Fig. 13. For loads $P>P_{ph}$, two types of response are observed according to the loading parameters $P$ and $t_r$. For the first type, the maximum dynamic displacement is greater than the static displacement while, for the second one, $u_{\text{max}}$ is very close to $u_{\text{stat}}$. Examples of both response types are presented below. For each of them, the dynamic curve representing the time evolution of the
displacement \( u_{\text{dyn}}(t) \) is compared to the static curve \( u_{\text{stat}}(t) \), which represents the evolution of the displacement, dynamic amplification being neglected. Accordingly, \( u_{\text{stat}}(t^*) \) is the static displacement associated with the value of the applied load \( P(t^*) \) at the time \( t^* \).

A response of type 1 (Fig. 14) is met when the system yields and gets beyond the static displacement corresponding to the final load \( P \). Then, it finally oscillates around a value of the displacement greater than this static displacement. If a behaviour of type 2 occurs (Fig. 15), then, when the plastic mechanism forms and the displacement suddenly increases, the latter however remains smaller than the static displacement corresponding to the final force \( P \). Next, the dynamic curve \( u_{\text{dyn}}(t) \) oscillates around a more or less constant value whilst the applied load continues to rise. Once the force \( P(t) \) has increased enough so that the associated static displacement meets the dynamic displacement, the latter starts to increase again, oscillating around the static curve. Eventually, the maximum dynamic displacement is close to \( u_{\text{stat}}(P) \).

The time evolution of the displacement for both response types can be explained as follows. When the plastic mechanism forms, the displacement rapidly increases and a distinct change in the slope of the curve \( u_{\text{dyn}}(t) \) is observed. However, due to its inertia, the system gets on the move progressively and the displacement remains at the beginning below the static displacement \( u_{\text{stat}}(t) \) corresponding to the applied load \( P(t) \). The system starts to accelerate and the dynamic displacement gets closer to the static one. Then it exceeds the latter and, the displacement of the system becoming higher than the static displacement associated with the force \( P(t) \) applied at the considered time, the velocity begins to decrease. The reduction of the velocity to zero, which corresponds to the first maximum of the dynamic displacement curve and a “stabilisation” of the system, may occur for a displacement smaller or greater than the static displacement associated with the
final load; that is what distinguishes the two behaviour types. Then, there is a sort of plateau in the curve \( u_{\text{dyn}}(t) \). In the first case (type 1), this plateau is infinite. In the second case (type 2), it carries on until the applied force \( P(t) \) has sufficiently increased so that the corresponding static displacement is equal to the dynamic displacement. Next, the dynamic curve oscillates around the static one to finally stabilize around a value of the displacement close to \( u_{\text{stat}}(P) \).

As far as internal forces are concerned, the axial load in the beams remains very small before the appearance of the three plastic hinges. When the moment in the middle and at the extremities of the double-beam reaches the plastic value \( M_{\text{pl}} \), the mechanism forms and the displacement rapidly increases, which induces the development of significant membrane forces in the beams. As a consequence, the moment acting in the plastic hinges decreases to respect the \( M-N \) plastic interaction relation. At the end, oscillations of \( M \) and \( N \) are observed while the displacement is oscillating around a constant value. The amplitude of the oscillations of the tension force is limited as the amplitude of the variations of the displacement is also small. On the other hand, the moment varies more importantly, in phase with the oscillations of \( u \). There corresponds a succession of elastic “unloadings-loadings” that can be observed on the \( M-N \) interaction diagram as well as on the \( M-u \) and \( N-u \) curves [7]. Moreover, it is interesting to note that the maximum axial load in the beams, which is obtained when \( u_{\text{max}} \) is reached, is the same as the tension force that would develop if this displacement was reached statically. Accordingly, this membrane force can be deduced from the sole knowledge of \( u_{\text{max}} \) and the static response.

### 4.3 Simplified approach to estimate the maximum dynamic displacement

The objective was to develop a simplified method to estimate the maximum displacement reached for given loading conditions \((P, t_r/T)\) with \( P > P_{\text{pl}} \). Then, it would be possible to predict the required deformation capacity of the structural members as well as the tension force they should resist. In view of the aspect of the \((P, u_{\text{max}})\) curves (see Fig. 13), the idea was to approach the latter, beyond the plastic plateau, by approximate curves established as follows [7]: section 1 \( \equiv \) horizontal at the level of \( P_{\text{pl}} \); section 2 \( \equiv \) pseudo-static curve; section 3 \( \equiv \) vertical between the pseudo-static and the static curve, at the abscissa \( u = u_{\text{max}} \) at which the actual \((P, u_{\text{max}})\) curve joins the static curve; section 4 \( \equiv \) static curve. An example of such a curve is presented at Fig. 16 (“correct” approximate curve). For too low values of the ratio \( t_r/T \), the dynamic curve does not join the static one and sections 3 and 4 cannot be defined. It is also possible that section 2 does not exist.

![Fig. 13. Example of an approximate dynamic curve](image-url)
To be able to draw such an approximate curve, the value of $u_{\text{trans}}$ is still to determine. The point ($P_{\text{trans}}$, $u_{\text{trans}}$) at which the dynamic curve ($P$, $u_{\text{max}}$) associated with a given value of $t_r/T$ joins the static curve corresponds to a transition between the two types of response previously described. Indeed, we have $u_{\text{max}} > u_{\text{stat}}$ for $P < P_{\text{trans}}$ (type 1) and $u_{\text{max}} \approx u_{\text{stat}}$ for $P > P_{\text{trans}}$ (type 2). As explained before, the behaviour type is governed by the value of the displacement ($u_{\text{plateau}}$) when the velocity is reduced to zero for the first time after the formation of the plastic mechanism. In fact, type 1 corresponds to $u_{\text{plateau}} > u_{\text{stat}}(P)$ while type 2 is associated with $u_{\text{plateau}} < u_{\text{stat}}(P)$.

As a consequence, if $u_{\text{plateau}}$ could be evaluated for a given loading, then the approximate dynamic curve ($P$, $u_{\text{max}}$)$_{\text{appr.}}$ corresponding to a fixed value of $t_r$ (or $t_r/T$) could be established following this procedure: (i) determination of the displacement $u_{\text{plateau}}$ for different values of $P$ and comparison with the static displacement $u_{\text{stat}}(P)$; (ii) identification of the force for which $u_{\text{plateau}} = u_{\text{stat}}(P)$; (iii) deduction of $u_{\text{trans}} = u_{\text{stat}}(P_{\text{trans}})$ from the static curve; (iv) drawing of the complete curve ($P$, $u_{\text{max}}$)$_{\text{appr.}}$. In order to carry out the first stage of this procedure, a simplified model was developed to estimate $u_{\text{plateau}}$ [3]. The latter is described below.

At first, a basic simplified model was developed under the following assumptions. It is a rigid-plastic model, in which the beams are considered to be infinitely rigid and thus keeping a constant length $l = l_0$. The plastic hinges developing at their extremities are submitted to a moment $M = M_{pl}$ assumed to be constant, interaction with the axial load being neglected. Finally, moderate displacements are supposed, which means that:

\[
\begin{align*}
\theta &\approx \frac{u}{l} (\approx \sin \theta \approx \tan \theta) \\
\cos \theta &\approx 1 - \frac{\theta^2}{2} \approx 1 - \frac{u^2}{2l^2}
\end{align*}
\]

\[\text{Fig. 17. Considered system and main definitions}\]

An energy equation was written, consisting in expressing that the work done by the external force $P(t)$ is equal to the sum of the kinetic energy, the work of the plastic hinges and the energy stocked in the lateral spring:

\[E_{\text{kine}} + E_{\text{hinges}} + E_{\text{spring}} = W_e \iff \frac{1}{2} M_{g} \ddot{u}^2 + \int M(\theta) \, d\theta + \int F_K(\delta_K) \, d\delta_K - \int P(u) \, du \]

\[\iff M_{g} \ddot{u}(t) + \frac{4M_{pl}}{l} + \frac{2K}{l^2} \cdot u(t)^3 = P(t) = P \frac{t}{t_r}
\]

Where $M_{g} = 1/3 \cdot m \cdot 2l_0 = 1/3 \cdot M_{tot}$ is the generalised mass of the system, $\delta_K$ is the elongation of the horizontal spring and $F_K$ the force it sustains.

This equation is only valid until the first maximum of the displacement is reached, which is $u_{\text{plateau}}$, and provided it occurs before the applied load become constant, so that it is expressed as $P(t) = P \cdot t/t_r$. However, these restrictions are of no consequence here. Indeed,
what we are interested in is the determination of $u_{\text{plateau}}$ and what happens after is no concern. Moreover, as the final objective is the determination of $P_{\text{trans}}$, only responses relatively close to the intermediate situation between the two behaviour types are interesting; and, in such cases, the plateau always starts at a time $t_{\text{plateau}} < t_r$. In order to resolve the previous equation, initial conditions have to be defined. In the considered rigid-plastic system, the displacement and the velocity are both zero until the plastic mechanism is formed. So the equation is resolved from the time $t_{pl}$, with the initial conditions: $u_0 = u(t_{pl}) = 0$ and $\dot{u}_0 = \dot{u}(t_{pl}) = 0$.

Unfortunately, this equation has no analytical solution and had to be numerically resolved. Moreover, it was observed that the use of this basic model leads to underestimate $u_{\text{plateau}}$, and then the value of $u_{\text{trans}}$ (see the corresponding curve on the graph of Fig. 16). That can be explained by the fact that different aspects neglected in the development of the basic model would induce greater displacements if taken into account. Eventually, the final model was developed from equation (1) but considering the M-N plastic interaction ($M_{pl}(N) = M_{pl}(u) < M_{p0}$) and the elongation of the beams ($l(N) = l(u) > l_0$). Then, the last approximate curve of Fig. 16 was drawn using this final model and following the previously described procedure. It is observed that the developed simplified method still leads to underestimate the extreme dynamic displacement for values of the force $P$ close to $P_{\text{trans}}$. For the considered example, the maximum unsafe error is about 8%.

5 CONCLUSIONS

At Liège University, the exceptional scenario “loss of a column” in a building structure has been under investigation for a few years with the final objective to propose design requirements to ensure an appropriate robustness of structures under the considered scenario.

The present paper gives a global overview of the adopted development strategy for this scenario, of the achievements in this field so far and of the ongoing research activities. In particular, simplified analytical methods have been developed to predict the static response of 2D steel and composite frames further to a column loss. Investigations are presently in progress to extend these methods to 3D structures. Besides, the dynamic behaviour of 2D structures has been investigated and a procedure has been developed to predict the dynamic response of a simplified substructure. There further validation and there extension to 3D structures have still to be developed further.
REFERENCES


