Disproportionate collapse analysis of steel buildings – a plastic limit approach on robustness

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1. ABSTRACT

Disproportionate collapse of building structures is defined as the partial or total failure of a building following a triggering event of local failure which cannot be absorbed through the internal continuity and ductility of the structural system of a building structure. As a consequence of the initial local failure or damage, a chain of new failures is propagated horizontally or vertically in the structural system, developing into the partial or total failure of the building, such that the final damage is disproportionate to the local initial failure caused by the triggering event. In that sense, disproportionate structural collapse occurs in a steel structure when a triggering event causes the full or partial collapse of the structural system disproportionally to the locality of the initiating event.

This paper presents a linear programming technique for the computation of respective collapse loads of steel building structures in the spirit of limit and shakedown analysis for damaged structures ([1]). Simple damage Kachanov indexes are introduced for elements which are considered as fully or partially damaged. Additionally, global robustness measures are proposed measuring the frame's residual bearing capacity after structural damage; the measures are defined as the ratios of safety factors of the damaged frame over that of the undamaged one.

2. INTRODUCTION

Disproportionate collapse can be provoked by numerous sources including construction or design flaws which surpass the common design base of current codes. Triggering events can be abnormal loads not included in the design, for example gas explosions, vehicle impacts, fire or extreme environmental loads which push the structural system well beyond the strength envelope. In this framework, all buildings are vulnerable to disproportionate collapse in some level. Currently, available structural codes are among the tools used by structural engineers in order to manage the level of risk in favor of public security. So far, relevant guidelines inside the codes refer to loads and strengths, addressing the risk in structural performance, without taking into account disproportionate collapse events and consequences. However, disproportionate collapse, although a rare event, inevitably awakes the public interest, mainly due to its sudden and unexpected characteristic, as well as its catastrophic human or financial consequences. Thus, the need for a relevant code framework is increasingly important.

For the computational assessment of the collapse load of damaged structures, damage-modified versions of common linearized yield criteria from design codes are used, rendering the optimization problem to be a linear programming one. Both the standard collapse load problem and the first plastification problem are considered, in order to capture ductile and non-ductile failures. The FEM data are obtained on the basis of standard 3node Timoshenko beam-column isoparametric elements and the solution of the linear programming problems is obtained by the commercial linear programming software MOSEK. Following this procedure, the collapse load factor is computed through the so-called direct methods of plasticity coupled with FEM. The computational realization is based on a combination of a linear FEM research code with linear programming software ([2]). The main objective of the present work is the quantification of robustness of steel structures using the direct methods of plasticity and the utilization of already accepted methods of disproportionate collapse analysis such as the alternate load path method of the DoD ([3]). The originality of the work lies on the computational approach regarding the quantification of the problem of disproportionate collapse in its general form and the definition of global robustness measures.

3. DISPROPORTIONATE COLLAPSE BASICS – ALTERNATE LOAD PATH METHOD

A central issue of AP method consists in the definition of the load cases to be considered. DoD makes a clear distinction between two separate load case categories: the deformation controlled actions and the force controlled actions. The intention for the classification of the load cases regards the type of failure of the elements associated with each one of the types of actions. Thus, deformation controlled actions concern rather ductile failures, while force controlled actions concern more brittle types of failure. First, a set of four basic load combinations $\varphi_{\mathrm{B}}^{(\mathrm{i})}, \mathrm{i} = 1, \ldots, 4$ is defined and let B be the respective index set, $B = \{1,2,3,4\}$. These four basic load combinations have a global character for the building - without any consideration of column removal and respective effects - and they are used for both deformation and force controlled actions. Assuming that the k-column is removed, each basic combination $\varphi_{\mathrm{B}}^{(\mathrm{i})}, \mathrm{i} \in \mathrm{B}$ generates the respective actual load cases $\varphi_{\mathrm{D}}^{(\mathrm{i})}(k)$ for deformation-controlled actions and $\varphi_{\mathrm{F}}^{(\mathrm{i})}(k)$ for force-controlled actions:

$$\varphi_{\mathrm{D}}^{(\mathrm{i})}(k) = \varphi_B^i + (\Omega_D(k) - 1)\Delta\varphi_B^i(k), \qquad \varphi_F^{(\mathrm{i})}(k) = \varphi_B^i + (\Omega_F - 1)\Delta\varphi_B^i(k) \tag{1}$$

where $\Delta\varphi_B^i(k)$ is simply the part of φ_B^i, which corresponds to the floor areas above the line of the removed k-th column. Factors $\Omega_D(k)$ and Ω_F are load increase factors (LIFs), which represent dynamic amplification phenomena, depending on the type of the structure's material and are mainly intended for linear static analysis. If the material is steel, Ω_D depends on the type of the connections between the elements above the removed k-th column and this fact is represented by the explicit dependence of this factor on the removed column k. In contrast, factor Ω_F is connection-independent. For nonlinear static analysis, the DoD suggests the use of a dynamic increase factor Ω_N and therefore eq. (1) takes the following form. Remarkably, Ω_N is always a smaller number than Ω_D.

$$\varphi_{\mathrm{D}}^{(\mathrm{i})}(k) = \varphi_B^i + (\Omega_N(k) - 1)\Delta\varphi_B^i(k) \tag{2}$$

4. LIMIT ANALYSIS OF DAMAGED STEEL FRAMES

In this section, a simple formulation of the limit analysis problem of damaged steel frames is described. Notation follows that of [4], [5], [6], [7].

4.1 Linear FEM analysis of damaged structures

Let Θ be a damaged structure, discretized within a geometrically linear FEM framework with NU free nodal displacements. Θ contains NG numerical integration points (Gauss points), which will be used in this work as stress checking points. Local quantities, always referred to the j-th Gauss point of Θ, are denoted by the respective subscript. Damage is introduced via indexes δ_j satisfying:

$$0 \leq \delta_j \leq 1, \quad j = 1, \dots \dots . NG_{\mathrm{D}}^{(\mathrm{i})} \tag{3}$$

which determine the local damage degree. Lower bound $\delta_j = 0$ is the local intact state condition (no damage) and upper bound δ_j=1 characterizes the state of full local damage. An element is removed, if the full-damage condition holds for all its Gauss points. Let **δ** be the vector of dimension NG, which lists all δ_j of the structure. The NU x NU linear stiffness matrix **K** of the damaged structure reads:

$$\mathbf{K} = \sum_{j=1}^{\mathrm{NG}} w_j \boldsymbol{B}_j^T \boldsymbol{C}_j^d \boldsymbol{B}_j \quad with\ \boldsymbol{C}_j^d = (1 - \delta_j)\boldsymbol{C}_j \tag{4}$$

where the intact quantities w_j, $\boldsymbol{C}_j$ and $\boldsymbol{B}_j$ are the standard integration weight, the modulus matrix and the strain-displacement matrix. In the sequel, the equilibrium matrices $\boldsymbol{H}_j = w_j \boldsymbol{B}_j^T$ will be also used. The linear displacement nodal vector **u** and the local elastic stress vectors $s_j^{(el)}$ under some external nodal load vector φ are obtained by the usual FEM equations:

$$\mathbf{Ku} = \varphi, \quad s_j^{(el)} = \boldsymbol{C}_j^d \boldsymbol{B}_j \boldsymbol{u}, \qquad j = 1, \dots, NG \tag{5}$$

In limit analysis, loading consists of a permanent nodal load vector φ_p and of a monotonically growing variable one $a\varphi_v$, where scalar a is the load pattern multiplier. Pair (φ_p, φ_v) constitutes the underlying load case and let $\boldsymbol{p}_j, \boldsymbol{v}_j$ be the elastic stresses due to φ_p, φ_v obtained by Eq.(5).

4.2 Piece-wise linear yield criteria of damaged sections

Local elastoplastic stresses under loading $\varphi_p + a\varphi_v$ can be written as the sum of the elastic ones and of the self-equilibrating stresses $\boldsymbol{\rho}_j$. In 3D frame analysis, s_j contains the axial/shearing forces and the twisting/bending moments:

$$s_j = \boldsymbol{p}_j + a\boldsymbol{v}_j + \boldsymbol{\rho}_j = (N, V_y, V_z, M_t, M_y, M_z)^T \tag{6}$$

and let us collect the respective individual intact plastic capacities $N_{pl}, V_{pl.y}, V_{pl.z}, M_{pl.t}, M_{pl.y}\ and\ M_{pl.z}$ in the diagonal entries of a local 6x6 diagonal matrix $\boldsymbol{N}_j$. The components of s_j have to satisfy the local yield criteria, modified by damage, which comprise the individual bounds posed by the individual capacities and the plastic interaction conditions. In this work, $N - M_y - M_z$ interactions are considered, which incorporate the respective individual capacity bounds. Correspondingly, the following partitioning of s_j is appropriate, where the dimensionless subvectors $\boldsymbol{y_i}, \boldsymbol{z_i}$ of s_j read:

$$s_j = \boldsymbol{N}_j(\boldsymbol{P_y y_i} + \boldsymbol{P_z z_i}), \qquad \boldsymbol{y}_j = (m_y, m_z, n)^T, \qquad \boldsymbol{z}_j = (v_y, v_z, m_t)^T \tag{7}$$

and $\boldsymbol{P_y}, \boldsymbol{P_z}$ are appropriate permutation matrices [7]. The yield criteria of the damaged j-section are now written in the partitioned form:

$$\boldsymbol{y}_j \in F_j, \qquad \boldsymbol{z}_j \in C_j \tag{8}$$

where the non-intecactively part $\boldsymbol{z}_j$ satisfies the individual plastic capacity bounds and the interactive part $\boldsymbol{y}_j$ must lie within a bounded polyhedron with MJ facets:

$$C_j = \{\boldsymbol{z} \in R^3 : |z_k| \leq (1 - \delta_j)\}, \qquad F_j = \{\boldsymbol{y} \in R^3 : \boldsymbol{L_j^T y} \leq (1 - \delta_j)\boldsymbol{\kappa}_j\} \tag{9}$$

where the MJ columns of $\boldsymbol{L}_j$ are the unit normals to the polyhedron faces and the intact capacity vector $\boldsymbol{\kappa}_j$ lists the respective positive distances of the facets to the origin. In this work, two types of F_j, are considered. First set $F_j^{(AISC)}$ contains the damage-modified AISC plastic interaction relations with MJ = 16 facets:

$$\begin{aligned} AISC: \ |n| + (8/9)(|m_y| + |m_z|) \quad &\leq \quad 1 - \delta_j, \\ 0.5|n| + |m_y| + |m_z| \quad &\leq \quad 1 - \delta_j \end{aligned} \tag{10}$$

and the second criterion $F_j^{(Rhomb)}$ j is a rhombic one with MJ = 8 facets:.

$$Rhombic\ (EC3):\ |n| + |m_y| + |m_z| \quad \leq \quad 1 - \delta_j, \tag{11}$$

whose intact form is accepted as a conservative interaction criterion dictated by EC3. It is noteworthy that, in the simple formulation of Eq.(8)-(11), damage enters only the capacity sides of all criteria inequalities. The criteria original intact form corresponds to $\delta_j = 0$. All criteria sets shrink to zero in case $\delta_j = 1$ (full damage).

4.3 Limit analysis and elastic limit problems

In a quasi-lower bound framework, the following optimization problem constitutes the limit analysis problem of the damaged frame Θ:

$$\begin{aligned} &P_{LMT}(\boldsymbol{\delta}, \varphi_p, \varphi_v) \quad Maximize \quad a \\ &subjected\ to:\ \sum^{\mathrm{NG}} \boldsymbol{H}_j \boldsymbol{s}_j = \varphi_p + a\varphi_v, \\ &\qquad s_j = \boldsymbol{N}_j(\boldsymbol{P_y y_i} + \boldsymbol{P_z z_i}), \quad \boldsymbol{y}_j \in F_j, \quad \boldsymbol{z}_j \in C_j, \quad j = 1, \dots, NG \end{aligned} \tag{12}$$

where problem dependence on the given damage vector $\boldsymbol{\delta}$ and on the underlying load case were made explicit for the purposes of the present paper. Using Eq.(8) yields the following, equivalent optimization problem:

$$\begin{aligned} &P_{LMT}(\boldsymbol{\delta}, \varphi_p, \varphi_v) \quad Maximize \quad a \\ &subjected\ to:\ \sum^{\mathrm{NG}} \boldsymbol{H}_j \boldsymbol{s}_j = 0, \\ &\boldsymbol{p}_j + a\boldsymbol{v}_j + \boldsymbol{\rho}_j = \boldsymbol{N}_j(\boldsymbol{P_y y_i} + \boldsymbol{P_z z_i}), \quad \boldsymbol{y}_j \in F_j, \quad \boldsymbol{z}_j \in C_j, \quad j = 1, \dots, NG \end{aligned} \tag{13}$$

Setting $\boldsymbol{\rho}_j = 0$ yields the elastic limit problem:

$$\begin{aligned} &P_{ELM}(\boldsymbol{\delta}, \varphi_p, \varphi_v) \quad Maximize \quad a \\ &subjected\ to:\ \sum^{\mathrm{NG}} \boldsymbol{H}_j \boldsymbol{s}_j = 0, \\ &\boldsymbol{p}_j + a\boldsymbol{v}_j = \boldsymbol{N}_j(\boldsymbol{P_y y_i} + \boldsymbol{P_z z_i}), \quad \boldsymbol{y}_j \in F_j, \quad \boldsymbol{z}_j \in C_j, \quad j = 1, \dots, NG \end{aligned} \tag{14}$$

which is the problem of first section plastification in frame Θ. P_{LMT} is a problem of elastoplasticity with unlimited ductility and it allows for non-elastic stress redistribution, represented by $\boldsymbol{\rho}_j$. Problem P_{ELM} can be used for situations with non-ductile behavior, since non-elastic stress redistribution within Θ is prohibited. In the last case, other intact capacity matrices $\boldsymbol{N}_j$ can be appropriate, e.g. by including buckling. From the numerical point of view, all maximization problems are linear programming problems, which can be solved by appropriate software. The partitioning, defined by Eq.(7), allows to take directly advantage of standard software options to treat separately simple variable bounds. Eq.(12) is preferable than Eq.(13), since the solution of Eq.(5) is avoided and damage appears only in the capacity sides of the criteria inequalities. Problem P_{ELM}, defined by Eq.(14), is a simple minmax problem. In the sequel, the safety factors obtained by the aforementioned limit analysis and elastic limit problems will be denoted by $a^*_{LMT}\left(\boldsymbol{\delta}, \varphi_p, \varphi_v\right)$ and $a^*_{ELM}\left(\boldsymbol{\delta}, \varphi_p, \varphi_v\right)$.

5. COLUMN REMOVAL: COLLAPSE ANALYSIS AND ROBUSTNESS

Collapse load analysis for column removal via limit analysis of damaged structures is now straightforward. Let K be the set of all columns, whose removal is to be considered. Let us assign to each column in K a global damage vector $\boldsymbol{\delta_k}$:

$$k \in K \qquad \rightarrow \qquad \boldsymbol{\delta_k} \in R^{NG} \tag{15}$$

This structural damage vector has all entries equal to zero (intact state) except those ones, which correspond to the Gauss points contained within the removed column: their damage indexes are set equal to one (full damage). Actually, Eq.(16) is far more general, since it encompasses partial damage. This way, the philosophy and scope of the AP method can be meaningfully extended by assigning a certain partial damage level to the columns, adjacent to the removed one. Let us, now, define the abbreviations:

$$a_{LMT}(i,0) = a^*_{LMT}\left(0,0,\varphi_B^{(i)}\right) \tag{16}$$

$$a_{ELM}(i,0) = a^*_{ELM}\left(0,0,\varphi_B^{(i)}\right) \tag{17}$$

$$a_{LMT}(i,k) = a^*_{LMT}\left(\boldsymbol{\delta_k}, 0, \varphi_B^{(i)}\right) \tag{18}$$

$$a_{ELM}(i,k) = a^*_{ELM}\left(\boldsymbol{\delta_k}, 0, \varphi_B^{(i)}\right) \tag{19}$$

$$A_{LMT}(i) = \min_{k \in K} a_{LMT}(i,k) \tag{20}$$

$$A_{ELM}(i) = \min_{k \in K} a_{ELM}(i,k) \tag{21}$$

Evidently, Eqs.(16)-(17) concern the undamaged structure (fully intact without any column removal effect). Condition:

$$A_{LMT}(i) \geq 1 \tag{22}$$

represents the "collapse survival" of the structure for all column removals under the i-th deformation controlled loading. Respectively, the "first plastification" condition of the damaged structure reads:

$$A_{ELM}(i) \geq 1 \tag{23}$$

By virtue of the safety factor definition, $Safety\ factor = \frac{Bearing\ capacity}{Load}$, the load multipliers a_{LMT}, a_{ELM} are structural bearing capacities measured in terms of the acting load. In this context, the ratios:

$$r_{LMT}(i,k) = a_{LMT}(i,k)/a_{LMT}(i,0), \qquad r_{ELM}(i,k) = a_{ELM}(i,k)/a_{ELM}(i,0) \tag{24}$$

Both these dimensionless, residual bearing capacities satisfy the condition, $0 \leq r \leq 1$ and the respective bearing capacity loss equals $1 - r$.

In this work, quantities $r_{LMT}(i,k)$ and $r_{ELM}(i,k)$ are proposed as respective robustness measures. The maximal possible value r = 1 indicates full robustness (bearing capacity loss equal to zero) and, respectively, the minimal possible value r = 0 characterizes zero robustness (capacity loss maximized). Obviously, the critical robustness measures, which correspond to $A_{LMT}(i)$ and $A_{ELM}(i)$ under the i-th loading, are given by:

$$R_{LMT}(i) \quad = \quad A_{LMT}(i)/a_{LMT}(i,0) = \min_{k\in K} r_{LMT}(i,k) \tag{25}$$
$$R_{ELM}(i) \quad = \quad A_{ELM}(i)/a_{ELM}(i,0) = \min_{k\in K} r_{ELM}(i,k) \tag{26}$$

The ranges of safety factors are completely described by defining the respective upper values:

$$\bar{A}_{LMT}(i) = \max_{k\in K} a_{LMT}(i,k), \qquad \bar{A}_{ELM}(i) = \max_{k\in K} a_{ELM}(i,k) \tag{27}$$

and the respective robustness measures are given by:

$$\bar{R}_{LMT}(i) = \bar{A}_{LMT}(i)/a_{LMT}(i,0) = \max_{k\in K} r_{LMT}(i,k) \tag{28}$$
$$\bar{R}_{ELM}(i) = \bar{A}_{ELM}(i)/a_{ELM}(i,0) = \max_{k\in K} r_{ELM}(i,k) \tag{29}$$

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ΕΛΛΗΝΙΚΗ ΠΕΡΙΛΗΨΗ

Η δυσαναλογική κατάρρευση ενός κτιρίου ορίζεται ως η καταστροφική μερική ή ολική αστοχία του κτιρίου ως επακόλουθο ενός γενεσιουργού γεγονότος τοπικής βλάβης, η οποία δεν μπορεί να απορροφηθεί από την εσωτερική συνέχεια και πλαστιμότητα του στατικού συστήματος του κτιρίου. Ως απόρροια της αρχικής τοπικής αστοχίας ή βλάβης, μία αλυσίδα νέων αστοχιών προκαλείται οριζοντίως ή κατακορύφως μέσα στο στατικό σύστημα, εξελισσόμενη σε μία εκτεταμένη μερική ή ολική αστοχία του κτιρίου, έτσι ώστε η τελικώς παραγόμενη βλάβη είναι δυσανάλογη της τοπικής αστοχίας που προκλήθηκε από το αρχικό γεγονός.

Η παρούσα εργασία παρουσιάζει τεχνικές μαθηματικού προγραμματισμού για τον υπολογισμό φορτίων κατάρρευσης μεταλλικών κτιρίων υπό βλάβη στο πνεύμα της οριακής και υπερωθητικής ανάλυσης ([1]). Απλοί δείκτες βλάβης Kachanov εισάγονται στα δομικά στοιχεία που μπορούν να υποστούν μερική ή ολική βλάβη. Επιπλέον, προτείνονται καθολικά μεγέθη στιβαρότητας καθοριζόμενα από την απομένουσα αντοχή των φορέων μετά από βλάβη. Τα μεγέθη αυτά ορίζονται ως οι λόγοι των συντελεστών ασφαλείας του φορέα υπό βλάβη προς τους συντελεστές ασφαλείας του ανέπαφου φορέα.