AN ADVANCED COMPUTATIONAL TOOL FOR INELASTIC ANALYSIS OF STEEL STRUCTURES

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1. ABSTRACT

In this paper an advanced computational tool for the inelastic analysis of steel structures accounting for axial-shear–flexure interaction, is presented. The proposed formulation is based on Boundary Element Method (BEM). The steel member is subjected to arbitrarily distributed or concentrated vertical loading along its length, while its edges are subjected to the most general boundary conditions. A displacement based procedure is employed and inelastic redistribution is modeled through a distributed plasticity model exploiting material constitutive laws and numerical integration over the cross-sections. An incremental - iterative solution strategy along with an efficient iterative process are employed, while the arising boundary value problem is solved employing the boundary element method. The proposed computational tool is employed for the analysis of representative numerical applications, illustrating its efficiency and accuracy.

2. INTRODUCTION

Design of steel structures based on elastic analysis are most likely to be extremely conservative not only due to significant difference between initial yield and full plastification in a cross section, but also due to the unaccounted for yet significant reserves of strength that are not mobilized in redundant members until after inelastic redistribution takes place. Thus, material nonlinearity is important for investigating the ultimate strength of a steel member that resists bending loading, while distributed plasticity models are acknowledged in the literature [1-3] to capture more rigorously material nonlinearities than cross sectional stress resultant approaches [4] or lumped plasticity idealizations [5, 6].

In this paper, a boundary element method is developed for the inelastic analysis of steel members accounting for axial-shear–flexure interaction. The essential features and novel aspects of the present formulation compared with previous ones are summarized as follows.

i. Axial, shear and flexure interaction is incorporated in this formulation.
ii. The shear deformation effect in the steel member is taken into account while shear locking is avoided by employing the same order of approximation for both the rotation due to bending and the derivative of the deflection.
iii. The formulation is a displacement based one taking into account inelastic redistribution along the member axis by exploiting material constitutive laws and numerical integration over the cross sections (distributed plasticity approach).
iv. An incremental - iterative solution strategy is adopted to restore global equilibrium of the beam.
v. The beam is supported by the most general nonlinear boundary conditions including elastic support or restrain.
vi. To the authors' knowledge, a BEM approach has not yet been used for the solution of the aforementioned problem, while the developed procedure retains most of the advantages of a BEM solution even though domain discretization is required.

Numerical results are worked out to illustrate the method, demonstrate its efficiency and accuracy.

3. STATEMENT OF THE PROBLEM

Let us consider a steel member of length l of arbitrary constant cross-section having at least one axis of symmetry (z-axis), occupying the two dimensional multiply connected region Ω of the y,z plane bounded by the $\Gamma_j\left(j=1,2,...,K\right)$ boundary curves, which are piecewise smooth, i.e. they may have a finite number of corners. The normal stress-strain relationship of the material is assumed to be elastic-plastic-strain hardening with initial modulus of elasticity E_0, shear modulus G, post-yield modulus of elasticity E_t, yield stress σ_{Y0} and yield strain ε_{Y0}. The member is subjected to the combined action of arbitrarily distributed or concentrated transverse loading $p_z\left(x\right)$ and bending moment $m_y\left(x\right)$ acting in the x direction.

3.1 Equations of global equilibrium

To establish global equilibrium equations, the principle of virtual work neglecting body forces is employed, that is

$$\int_V \left(S_{xx}\delta\varepsilon_{xx}+S_{xz}\delta\gamma_{xz}\right)dV=\int_F \left(t_x\delta u+t_z\delta w\right)dF \tag{1}$$

where the integral quantities represent the strain energy and the external load work while $\delta\left(\cdot\right)$ denotes virtual quantities, V is the volume and F is the surface of the member. After conducting some algebraic manipulations, the global equilibrium equations are obtained as

$$EA\left(u''+w'w''\right)+\frac{\partial N^{pl}}{\partial x}=-p_x \Rightarrow \frac{d\left(N^{el}+N^{pl}\right)}{dx}=\frac{dN}{dx}=-p_x \tag{2}$$

$$EI_y\theta_y''+\frac{\partial M^{pl}}{\partial x}-GA_z(z)\left(w'+\theta_y\right)-Q_z^{pl}=-m_y \Rightarrow \frac{dM}{dx}-Q_z=-m_y \tag{3}$$

$$\underbrace{EA\left(\left(u'+\frac{1}{2}w'^2\right)w'\right)'+\frac{\partial\left(N^{pl}w'\right)}{\partial x}}_{\frac{d(Nw')}{dx}}+\underbrace{GA_z(z)\left(w'+\theta_y\right)'+\frac{\partial Q_z^{pl}}{\partial x}}_{\frac{dQ_z}{dx}}=-p_z \tag{4}$$

along with its corresponding boundary conditions

$$a_1u+a_2\left(EA\left(u'+\frac{1}{2}w'^2\right)+N^{pl}\right)=a_3 \Rightarrow a_1u+a_2N=a_3 \tag{5}$$

$$b_1\theta_y+b_2\left(EI_y\theta_y'+M^{pl}\right)=b_3 \Rightarrow b_1\theta_y+b_2M_y=b_3 \tag{6}$$

$$\begin{aligned}&c_1w+c_2\left[EA\left(u'+\frac{1}{2}w'^2\right)w'+N^{pl}w'+GA_z(z)\left(w'+\theta_y\right)+Q_z^{pl}\right]=c_3\\ \Rightarrow\quad & c_1w+c_2V_z=c_3\end{aligned} \tag{7}$$

where u, w are the displacement components of the centroid , θ_y is the angle of rotation due to bending of the cross-section with respect to its centroid, N, Q_z, M_y are the stress resultant corresponding to the internal axial force, shear force and bending moment, respectively, while $a_i, b_i, c_i \ (i=1,2,3)$ are functions specified at the member ends.

3.2 Integral Representations – Numerical Solution

According to the precedent analysis, the inelastic problem of steel members reduces to establishing the displacement components satisfying the boundary value problem described by the governing differential eq. (2-4) along the boundary conditions (5-7). This boundary value problem is solved employing BEM [7], as this is developed in [8] for the solution of second order differential equation with constant coefficients, after some modifications.

4. NUMERICAL EXAMPLES

The influence of the axial-shear–flexure coupling on the behavior of the steel structures is investigated in this example. For this purpose, an I-shaped cross section beam of length $l=2m$, has been studied. The geometric properties of the selected cross section are given in Table 1, while the beam's material is considered to be elastic-perfectly plastic with modulus of elasticity $E=213.4GPa$, shear modulus

$G = 82GPa$ and yielding stress $\sigma_y = 285MPa$. The beam is either clamed of fixed-pinned supported, while it is subjected to monotonically increasing uniformly distributed load. The beam is discretized with 22 linear longitudinal elements, 43 quadrilateral cells (12 layers in the wed and 2 in each flange) and a 1×1 Gauss integration scheme for each cell.

Total height	$h = 0.3m$	Flange width	$t_f = 0.02m$
Total width	$b = 0.3m$	Web width	$t_w = 0.01m$
Moment of Inertia	$I_y = 25.0247\times10^{-5}m^4$	Shear Correction Factor	$a_z = 5.3897$

Table 1. Geometric properties of the I-shaped cross section

In Fig. 1(a,b) the load-displacement curves are presented, performing either geometrically linear or nonlinear analysis, for both the boundary condition cases. The results are compared with those obtained from a FEM model implemented in NX Nastran [9] by employing 2400 quadrilateral shell elements. Excellent convergence between the results is observed. In the same figures the von Mises stress σ_{vM} distribution is also presented illustrating the plastification of the wed, as well as the non-symmetry of the normal stresses due to the developed axial force. Additionally, the flexure-only response is presented in these figures. Since the beam yields in shear, the Euler-Bernoulli model fails to capture the nonlinear response and overestimates the collapse load of approximately 320% for the clamed and 256% for the fixed-pinned boundary conditions.

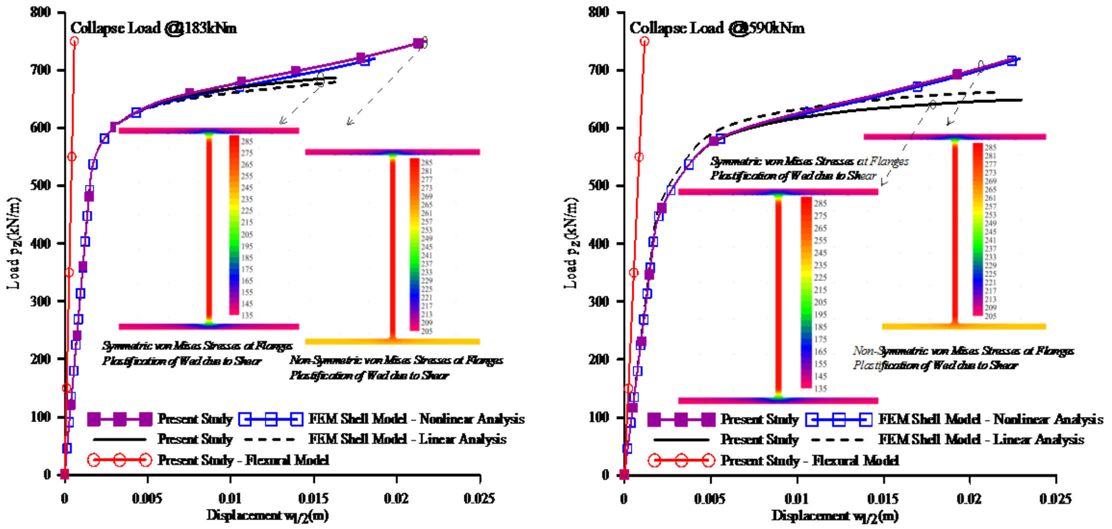

Fig 1. *Midpoint load–displacement curve of the clamed (a)and fixed-pinned (b) beam.*

The main reason for that divergence is its inability to predict the exact collapse mechanism. This can be also evident from the von Mises stress distribution contour diagram presented in Fig 2. In more detail, Fig. 2(a) show the stress distribution along the length of the web for geometrically nonlinear analysis as compared with those obtained from the shell model [9], while in Fig. 2 (b) the same results are resented for geometrically linear analysis. From this figure, the predominant shear character of the collapse mechanism is observed while the accuracy of the proposed beam formulation is verified.

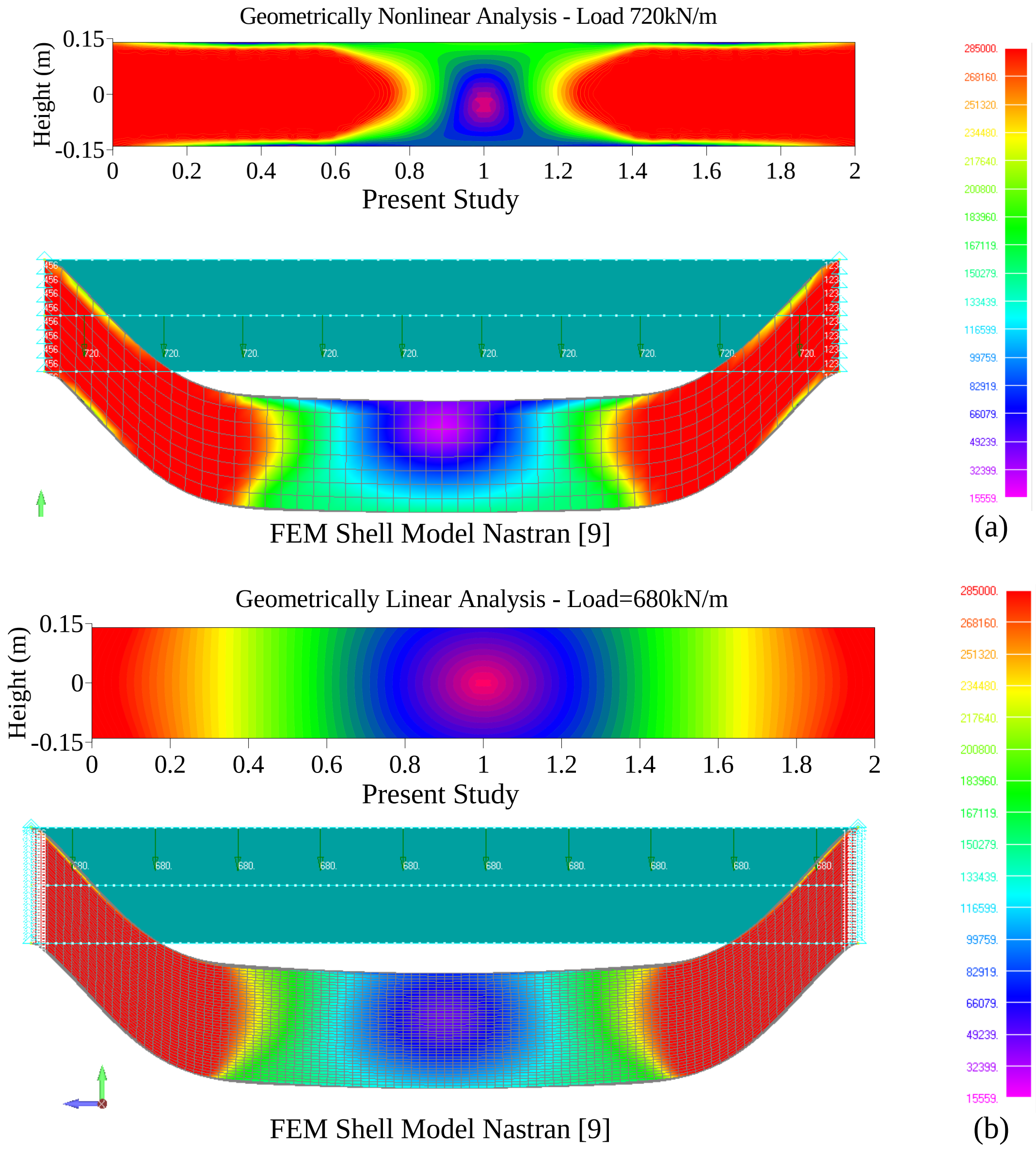

Fig.2 von Mises stress distribution contour diagrams along the length of the web for geometrically nonlinear (a) & linear (b) analysis.

5. CONCLUDING REMARKS

In this paper, BEM approach is developed for the inelastic analysis of steel members accounting for axial-shear–flexure interaction. The main conclusions that can be drawn from this investigation are:

a. The proposed beam formulation is capable of obtaining results of high accuracy, as verified by comparing with 2D/3D FEM models, with minimum computational cost. Its advantageous character over more refined approaches is also enhanced by the following:

- The developed beam formulation reduces significantly modeling effort (shell/solid models require cumbersome pre-processing even in relatively simple cases).
- It permits isolation of structural phenomena and results interpretation (quantities such as stress resultants etc. are also evaluated in contrast to shell/solid model which yields only displacements and stress components).
- It allows straightforward model handling (boundary conditions and external loading are easily simulated).
- It facilitates parametric analyses (solid modeling often requires construction of multiple models).

b) Accurate results are obtained using a relatively small number of nodal points across the longitudinal axis.
c) The interaction between shear and flexure is of paramount importance in the inelastic analysis of steel structures.
d) The influence of the geometrically nonlinear analysis is confirmed.

6. ACKNOWLEDGMENT

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ΣΤΟΙΧΕΙΟ ΔΟΚΟΥ ΓΙΑ ΠΡΟΧΩΡΗΜΕΝΗ ΑΝΕΛΑΣΤΙΚΗ ΑΝΑΛΥΣΗ ΚΑΤΑΣΚΕΥΩΝ ΑΠΟ ΧΑΛΥΒΑ

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ΠΕΡΙΛΗΨΗ

Στην παρούσα εργασία, παρουσιάζεται η μεθοδολογία για τη ανάπτυξη στοιχείου δοκού για την ανελαστική ανάλυση κατασκευών από χάλυβα, λαμβάνοντας υπόψη την αλληλεπίδραση αξονικής, τέμνουσας και καμπτικής ροπής. Η προτεινόμενη μεθοδολογία βασίζεται στη Μέθοδο Συνοριακών Στοιχείων (ΒΕΜ). Το δομικό στοιχείο, τυχούσης διατομής μονής συμμετρίας, υπόκειται στις πλέον γενικές μη γραμμικές συνοριακές συνθήκες, ενώ κατά την ανάλυση λαμβάνεται υπόψη το φαινόμενο διατμητικής παραμόρφωσης με τη βοήθεια της θεωρίας δοκού Timoshenko, η οποία συνυπολογίζει έμμεσα το φαινόμενο αυτό μέσω διορθωτικών συντελεστών διάτμησης. Το στοιχείο υπόκειται σε τυχαία κατανεμημένα ή/και συγκεντρωμένα αξονικά και εγκάρσια φορτία. Οι εξισώσεις ισορροπίας εξάγονται στην παραμορφωμένη κατάσταση συνυπολογίζοντας τη γεωμετρική μη γραμμικότητα λόγω των μετρίως μεγάλων μετατοπίσεων. Οι πλαστικές παραμορφώσεις προσδιορίζονται μέσω προσομοιώματος κατανεμημένης πλαστικότητας (distributed plasticity model) χρησιμοποιώντας τριδιάστατες καταστατικές σχέσεις. Οι σχέσεις αυτές ολοκληρώνονται με τη βοήθεια κατάλληλης επαναληπτικής μεθόδου. Η αποτελεσματικότητα και το εύρος εφαρμογής της μεθόδου παρουσιάζεται μέσα από παραδείγματα με ιδιαίτερο πρακτικό ενδιαφέρον.