A DEGRADING HYSTERETIC BEAM ELEMENT FOR STEEL STRUCTURES

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1. ABSTRACT

In this paper the hysteretic beam element proposed by Triantafyllou and Koumousis [1] is extended to account for stiffness degradation, strength deterioration and pinching phenomena. The behavior of the element is governed by the Bouc-Wen model of hysteresis while stiffness and strength degradation are based on Baber and Wen model [2] and pinching on Foliente's model [3]. The formulation is based on additional hysteretic degrees of freedom which are considered as hysteretic curvatures and hysteretic axial deformations of the cross-sections. The entire set of governing equations of the structure is solved simultaneously by converting the system into state space form. This consists of the linear global equations of motion and the nonlinear local constitutive evolutionary equations for every element. Furthermore, degradation phenomena (related to material, structural member and connection behavior) are treated in a unified manner and are easily controlled through the model parameters at the element level. Numerical results are presented which are compared with existing experimental data demonstrating the efficacy of the proposed element in the analysis of steel structures.

2. INTRODUCTION

Hysteresis is a phenomenon where a system's response depends not only on its current state but also on the history of previous states. It is a nonlinear phenomenon usually considered as rate-independent. Several hysteretic models have been developed in the past to address hysteresis which can be divided into two categories, the multi-segmental models and smooth models.

Multi-segmental models, such as bilinear, trilinear and other multi-linear models preceded the smooth ones and define the behavior in piecewise linear stages such as the initial elastic, yielding, hardening/softening stages with unloading and reloading branches. Examples of such models are those proposed by Clough (1966), Takeda (Takeda et al. 1970) [8] and Park (Park et al. 1987) [9] among others.

On the other hand, smooth hysteretic models are based on continuous-smooth change of stiffness after yield and can accommodate degradation phenomena. They are able to model different types of hysteretic behavior and are based on a smooth hysteretic function and a set of user defined parameters. Bouc-Wen model belongs to this category and has been used widely in different applications. Extensions of this model are the Baber – Noori (1985) [6], Baber – Wen (1981) [2], Foliente (1995) [3] and Sivaselvan-Reinhorn model (2000) [7] and more recently the one by Kottari et al. (2014) [12].

Under cyclic loading the phenomena of stiffness degradation, strength deterioration and pinching are usually manifested. These are caused by plastic regions being created and extended in later loading cycles together with local buckling regions, which lead to stiffness and strength loss. Pinching is the sudden loss of stiffness resulting in loops that are thinner in middle range than at the ends. It is caused by the loosening and slipping of joints in steel structures. In this paper the models proposed by Baber and Wen (1981) [2] are employed for stiffness and strength degradation together with the one by Foliente (1995) [3] for pinching.

3. THE HYSTERETIC BEAM ELEMENT WITH DEGRADATIONS

For the 2D hysteretic beam element the inelastic moment-curvature relation and axial force-axial centerline strain relation at a cross section at a distance s from the start node are expressed as follows:

$$\begin{aligned} M(s,t) &= aEI\varphi(s,t) + (1-a)EIz(s,t) \\ N(s,t) &= a_u EAu(s,t) + (1-a_u)EAz_u(s,t) \end{aligned} \tag{1}$$

The hysteretic parameter z is defined as the hysteretic part of the curvature regarding the bending degrees of freedom and z_u is the hysteretic part of the axial centerline deformation. The Bouc–Wen hysteretic differential equations with degradations are expressed as:

$$\begin{aligned} \dot{z}(t) &= h\left\{1 - v_s\left|\frac{z}{z_y}\right|^n (\beta + \gamma sgn(z\dot{\varphi}))\right\}\frac{\dot{\varphi}}{n_s} \\ \dot{z}_u(t) &= h_u\left\{1 - v_{s,u}\left|\frac{z_u}{\varepsilon_{0y}}\right|^n (\beta + \gamma sgn(z\dot{\varepsilon}_0))\right\}\frac{\dot{\varepsilon}_0}{n_{s,u}} \end{aligned} \tag{2}$$

where v_s is the parameter that controls the strength deterioration and n_s is the parameter controlling stiffness degradation and h is the parameter controlling pinching. These parameters are functions of the hysteretic energy dissipation (the energy dissipated by the hysteretic spring) and are defined by the following relations [3]:

$$\begin{aligned} n_s &= 1 + c_n e^h,\ \ c_n \geq 0 \\ v_s &= 1 + c_v e^h,\ \ c_v \geq 0 \end{aligned} \tag{3}$$

Additionally h is defined as:

$$h = h\left(z, sgn\left(du\right), e\right) = 1 - \zeta_1 \exp\left(-\frac{\left(zsgn\left(du\right) - qz_u\right)^2}{\zeta_2^{\ 2}} \right) \tag{4}$$

$$z_u = z_u\left(e\right) = \left(\frac{1}{\nu_s\left(\beta + \gamma\right)} \right)^{\frac{1}{n}} \tag{5}$$

$$\begin{aligned} \zeta_1 &= \zeta_1\left(e\right) = \zeta_{1,0}\left(1 - exp\left(-pe\right)\right) \\ \zeta_2 &= \zeta_2\left(e\right) = \left(\psi_0 + \delta_\psi e\right)\left(\lambda + \zeta_1\left(e\right)\right) \end{aligned} \tag{6}$$

Assuming Euler-Bernoulli beam theory curvature is approximated by:

$$\varphi = \frac{\partial^2 w}{\partial s^2} \tag{7}$$

where *w* is the transverse deflection of the beam. Substituting relation (7) into (1):

$$M\left(s,t\right) = EI\tilde{\varphi}(s,t) \qquad \tilde{\varphi}\left(s,t\right) = a\frac{\partial^2 w(s,t)}{\partial s^2} + (1-a)z(s,t) \tag{8}$$

where $\tilde{\varphi}(s,t)$ can be regarded as a measure of an “equivalent generalized curvature”.
Similarly considering the axial degrees of freedom:

$$N\left(s,t\right) = EA\tilde{\varepsilon}_0(s,t) \qquad \tilde{\varepsilon}_0\left(s,t\right) = a_u\frac{\partial u(s,t)}{\partial s} + (1-a_u)z_u(s,t) \tag{9}$$

where *N* is the axial force and $\tilde{\varepsilon}_0(s,t)$ is the generalized axial centerline strain.

3.1 DISCRETIZATION WITH FINITE ELEMENT METHOD

The displacement field is interpolated using cubic polynomial shape functions [7]:

$$\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_3 & N_4 & 0 & N_5 & N_6 \end{bmatrix}\left\{d\right\} \tag{10}$$

where the nodal displacement vector *{d}* is defined as $\left\{d\right\} = \left\{u_1 \ \ w_1 \ \ \theta_1 \ \ u_2 \ \ w_2 \ \ \theta_2\right\}^T$.
The total curvature φ can be expressed as:

$$\varphi = \left[0 \quad N_{3,ss} \quad N_{4,ss} \quad 0 \quad N_{5,ss} \quad N_{6,ss}\right]\left\{d\right\} = \left[B_b\left(s\right)\right]\left\{d\right\} \tag{11}$$

where the subscript *,ss* denotes double differentiation with respect to the space variable s, and results in:

$$\varphi(s) = \left[0 \quad -\frac{6}{L^2} + \frac{12s}{L^3} \quad -\frac{4}{L} + \frac{6s}{L^2} \quad 0 \quad \frac{6}{L^2} - \frac{12s}{L^3} \quad -\frac{2}{L} + \frac{6s}{L^2}\right]\left\{d\right\} = \left[B_b\left(s\right)\right]\left\{d\right\} \tag{12}$$

The hysteretic curvature is defined via the following linear shape functions [1] and can be written as:

$$z_\varphi = \left[N_7 \quad N_8\right]\begin{Bmatrix} z_1^b \\ z_2^b \end{Bmatrix} = \left[N\right]_z^b \begin{Bmatrix} z_1^b \\ z_2^b \end{Bmatrix} \qquad \mathrm{N}_7 = 1 - \frac{s}{L} \quad N_8 = \frac{s}{L} \tag{13}$$

the generalized curvature can therefore be expressed as:

$$\tilde{\varphi} = a\left[0 \quad N_{3,ss} \quad N_{4,ss} \quad 0 \quad N_{5,ss} \quad N_{6,ss}\right]\left\{d\right\} + (1-a)\left[N_7 \quad N_8\right]\begin{Bmatrix} z_1^b \\ z_2^b \end{Bmatrix} \tag{14}$$

Similarly the generalized centerline axial deformation is expressed as:

$$\tilde{\varepsilon}_0 = a\left[N_{1,s} \quad 0 \quad 0 \quad N_{2,s} \quad 0 \quad 0\right]\left\{d\right\} + (1-a)\left[N_9 \quad N_{10}\right]\begin{Bmatrix} z_1^u \\ z_2^u \end{Bmatrix} \tag{15}$$

where the corresponding shape functions are[1]:

$$N_9 = -\frac{1}{L} \quad N_{10} = \frac{1}{L} \tag{16}$$

3.3 CONSTITUTIVE MATRIX RELATION

By means of the principle of virtual work the following relation is obtained [1]:

$$\left[\begin{array}{cccccc:cccc}
\frac{a_u EA}{L} & 0 & 0 & -\frac{a_u EA}{L} & 0 & 0 & \frac{-(1-a_u)EA}{2} & \frac{-(1-a_u)EA}{2} & 0 & 0 \\
0 & \frac{12aEI}{L^3} & \frac{6aEI}{L^2} & 0 & -\frac{12aEI}{L^3} & \frac{6aEI}{L^2} & 0 & 0 & -\frac{(1-a)EI}{L} & \frac{(1-a)EI}{L} \\
0 & \frac{6aEI}{L^2} & \frac{4aEI}{L} & 0 & -\frac{6aEI}{L^2} & \frac{2aEI}{L} & 0 & 0 & -(1-a)EI & 0 \\
-\frac{a_u EA}{L} & 0 & 0 & \frac{a_u EA}{L} & 0 & 0 & \frac{-(1-a_u)EA}{2} & \frac{-(1-a_u)EA}{2} & 0 & 0 \\
0 & -\frac{12aEI}{L^3} & -\frac{6aEI}{L^2} & 0 & \frac{12aEI}{L^3} & -\frac{6aEI}{L^2} & 0 & 0 & \frac{(1-a)EI}{L} & -\frac{(1-a)EI}{L} \\
0 & \frac{6aEI}{L^2} & \frac{2aEI}{L} & 0 & -\frac{6aEI}{L^2} & \frac{4aEI}{L} & 0 & 0 & 0 & (1-a)EI
\end{array}\right]
\left\{\begin{array}{c} \{d\} \\ \hdashline \{z_u\} \\ \{z_b\} \end{array}\right\} = \left\{\begin{array}{c} N_1 \\ Q_1 \\ M_1 \\ N_2 \\ Q_2 \\ M_2 \end{array}\right\} \tag{17}$$

Relation (17) expresses the equilibrium including both the elastic and hysteretic behavior of the element, where the axial forces are uncoupled with bending moments and shearing forces not only in the elastic but also in hysteretic part. This relation can be written as:

$$\{\mathrm{f}\} = a[k]\{d\} + (1-a)[h]\{z\} \tag{18}$$

where the first term represents the elastic behavior based on the reduced, plastic stiffness and the second term adds the hysteretic part. These basic matrices are defined at elemental level, are formed once in the beginning of the analysis and remain unchanged thereafter. Transforming to the global system using the 2D transformation matrix [Λ] we get:

$$\{F\} = a[\Lambda]^T [k][\Lambda]\{u\} + (1-a)[\Lambda]^T [h]\{z\} \tag{19}$$

3.4 EVOLUTION EQUATIONS

The nonlinear behavior of the element is governed by the Bouc-Wen evolution equations (2) including stiffness, strength degradation and pinching. Equations (2) using (12) transformed to the global system (19) can be expressed as:

$$\dot{z}_b(s,t) = h\left(1 - v_s\left|\frac{z(s,t)}{z_y}\right|^n \left(\beta + \gamma sgn\left(z(s,t)\left[B_b(s)\right][\Lambda]\{\dot{u}\}\right)\right)\right)[B_b(s)][\Lambda]\frac{\{\dot{u}\}}{n_s} \tag{20}$$

$$\dot{z}_u(s,t) = h_u\left(1 - v_{s,u}\left|\frac{z_u(s,t)}{z_y}\right|^n \left(\beta + \gamma sgn\left(z_u(s,t)\left[B_u(s)\right][\Lambda]\{\dot{u}\}\right)\right)\right)[B_u(s)][\Lambda]\frac{\{\dot{u}\}}{n_{s,u}} \tag{21}$$

3.5 STATE SPACE FORMULATION

For the dynamic problem the equation of motion for a multi degree of freedom structure can be established as follows:

$$[M]_s\{\ddot{U}\} + [C]_s\{\dot{U}\} + [K]_s\{U\} + [H]_s\{Z\} = \{P(t)\} \tag{22}$$

where $[M]_s$ is the mass matrix, $[C]_s$ is the viscous damping matrix, $[K]_s$ is the stiffness matrix, containing only the elastic part of relation (17), $[H]_s$ is the hysteretic matrix of the structure and $\{P(t)\}$ is the vector of external forces. These matrices are assembled following the direct stiffness method [4], while the viscous damping matrix in general may be of the form of a Rayleigh damping matrix [5]. The hysteretic behavior is defined at the element level in terms of hysteretic curvatures and centerline axial deformations from relations (20) and (21). The hysteretic matrix of each element, expressed in the global system, is appended to form the corresponding hysteretic matrix of the structure. Equations

(22), together with evolution equations (20) and (21), fully describe the response of the system to a given external force and initial conditions. To solve the system of equations, this can be transformed into a set of first order differential equations in state space form. Introducing the vector of nodal velocities $\{\dot{u}\}$ as auxiliary unknown vector one can write the system the following form:

$$\{\dot{x}\} = G(\{x\}) + \{P(t)\} \tag{23}$$

where the vector {x} is defined as:

$$\{x\}^T = \left[\{U\}^T \quad \{\dot{U}\}^T \quad \{Z\}^T \right] \tag{24}$$

and G:

$$G(\{x\}) = \begin{bmatrix} 0 & I & 0 \\ [M]^{-1}[K] & [M]^{-1}[C] & [M]^{-1}[H] \\ 0 & Y(\{\dot{U}\},\{Z\}) & 0 \end{bmatrix} \tag{25}$$

The operator G is a state dependent operator since Y contains the evolution equations of every element:

$$Y_j^i\left(\{\dot{u}\}^i,\{z\}^i\right) = h\left(1 - v_s\left|\frac{z_j(t)}{z_y}\right|^n\left(\beta + \gamma\,\mathrm{sgn}\left(z_j(t)[B]_j[\Lambda]\{\dot{u}\}\right)\right)\right)[B]_j[\Lambda]\frac{\{\dot{u}\}^i}{n_s} \tag{26}$$

The above system, for specific dynamic loading is integrated using a variable-order solver based on the numerical differentiation formulas (NDFs) i.e. a multistep solver.

4. NUMERICAL RESULTS

To demonstrate the efficiency of the proposed element in simulating the hysteretic behaviour of steel structural members the results are compared with existing experimental data. In [10] a beam column connection is tested under continiously applied cyclic displacement. The geometry of the specimen can be seen in Fig. 1. Three beam elements were used for the analysis and the results are shown in Fig. 1 reagarding the total moment – plastic rotation relation of the column face where black line is for the experiment and blue line for the analysis.

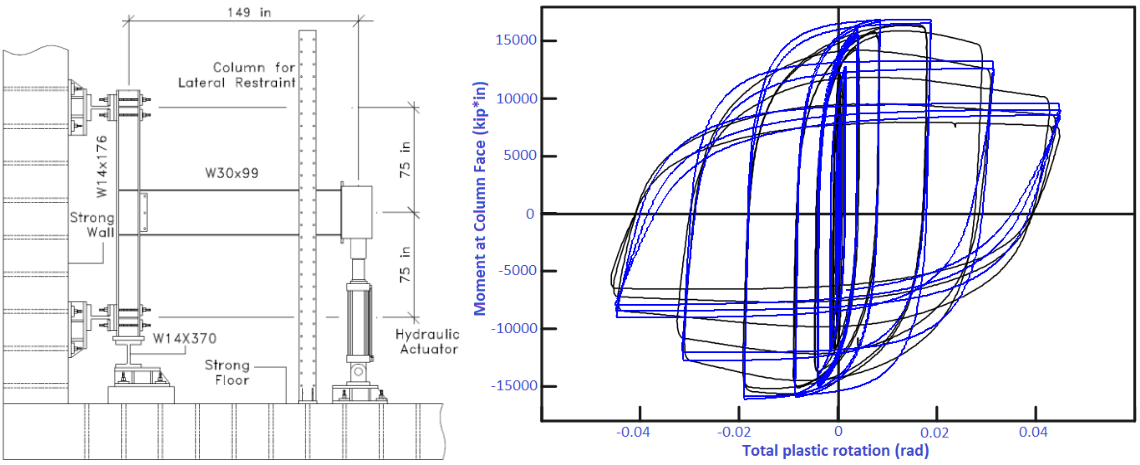

Fig. 1- Specimen LS- 1 (left) [10] and its analytical and experimental response (right).

In the next example a typical portal frame with span 24 ft and height 14 ft, and W27x114 columns and W24x94 beam under self-weight is subjected to the El Centro accelerogram, scaled up by a factor of 4. The yield limit is 36 ksi and the Bouc Wen parameters used are β=γ=0.5 and n=25. The results obtained for the displacements of the top of the frame are in good agreement against those obtained using OpenSees code [11] that employs multi segmental degrading models, which in general require significantly more computing time.

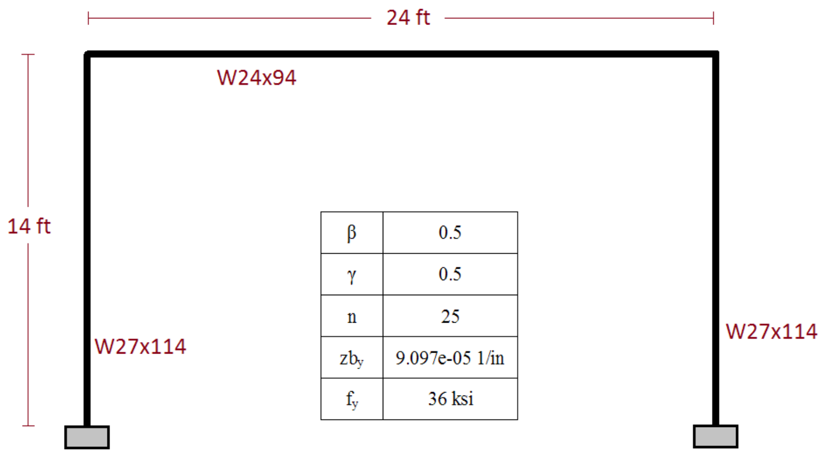

Fig. 2 – Portal frame geometry and materials.

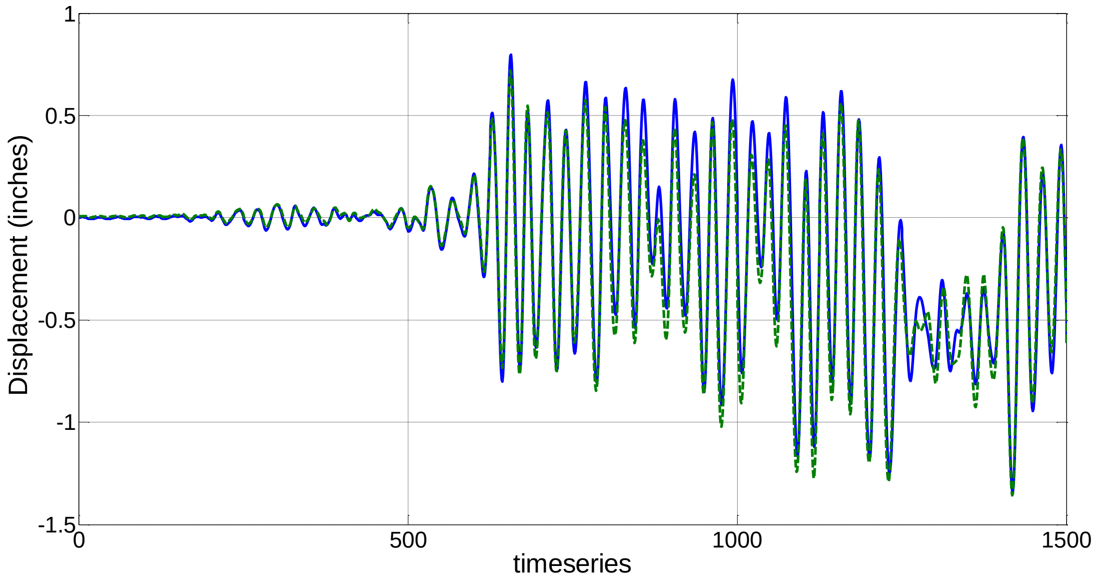

Fig. 3 - Displacement of the portal frame subjected to El Centro accelerogram.

5. CONCLUDING REMARKS

A wide range of hysteretic behavior can be modeled by the proper control of the Bouc-Wen model parameters and the degradation parameters. The beam element is formulated with four new degrees of freedom accounting for the hysteretic part of the curvature and the axial centerline deformation. The entire problem is casted into two sets of equations namely the linear equations of motion and the nonlinear evolution equations, which are solved simultaneously by implementing a numerical differentiation scheme. Comparisons with experimental data and other structural analysis code show good results demonstrating the effectiveness of the proposed model to simulate degradation effects in hysteretic behavior.

6. REFERENCES

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ΣΤΟΙΧΕΙΟ ΔΟΚΟΥ ΜΕ ΥΣΤΕΡΗΤΙΚΑ ΑΠΟΜΕΙΟΥΜΕΝΑ ΜΟΝΤΕΛΑ ΓΙΑ ΜΕΤΑΛΛΙΚΕΣ ΚΑΤΑΣΚΕΥΕΣ

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ΠΕΡΙΛΗΨΗ

Σε αυτό το άρθρο παρουσιάζεται ένα πεπερασμένο στοιχείο δοκού κατάλληλο για την ανελαστική δυναμική ανάλυση κατασκευών. Το υστερητικό στοιχείο δοκού που παρουσιάστηκε από τους Τριανταφύλλου και Κουμούσης επεκτείνεται ώστε να περιλάβει τα φαινόμενα της μείωσης της αντοχής, της δυσκαμψίας καθώς και το φαινόμενο της στένωσης. Η συμπεριφορά του στοιχείου καθορίζεται από το υστερητικό μοντέλο του Bouc–Wen, ενώ η απομείωση της δυσκαμψίας και της αντοχής βασίζονται στο μοντέλο των Baber και Wen και η στένωση στο μοντέλο του Foliente. Το προτεινόμενο στοιχείο διαμορφώνεται με επιπρόσθετους βαθμούς ελευθερίας, την υστερητική καμπυλότητα και τις υστερητικές αξονικές παραμορφώσεις. Τα μητρώα μορφώνονται με τη μέθοδο της άμεσης ακαμψίας, και το σύστημα των εξισώσεων μετατρέπεται σε μορφή χώρου κατάστασης για να επιλυθεί. Το σύστημα των εξισώσεων αποτελείται από τις εξισώσεις κίνησης καθώς και τις υστερητικές διαφορικές εξισώσεις Bouc–Wen με τις απομειώσεις. Η συμπεριφορά του στοιχείου (που σχετίζεται με το υλικό, το δομικό στοιχείο, τις συνδέσεις κ.ά.) ελέγχεται εύκολα από τη ρύθμιση των παραμέτρων του μοντέλου. Τέλος παρουσιάζονται παραδείγματα και συγκρίσεις του μοντέλου με πειραματικά δεδομένα που επιβεβαιώνουν τις δυνατότητές του.