DIRECT PERFORMANCE-BASED SEISMIC DESIGN OF A STEEL MOMENT-RESISTING FRAME USING YIELD FREQUENCY SPECTRA

Dimitrios Vamvatsikos
School of Civil Engineering
National Technical University of Athens, Greece
e-mail: divamva@mail.ntua.gr

Mark A. Aschheim
Department of Civil Engineering
Santa Clara University, California, USA
e-mail: maschheim@scu.edu

1. ABSTRACT

Yield Frequency Spectra (YFS) are employed to enable the direct seismic design of a 4-story steel moment-resisting frame subject to a set of performance objectives. YFS offer a unique view of the entire solution space for structural performance. This is measured in terms of the mean annual frequency (MAF) of exceeding arbitrary ductility (or displacement) thresholds, versus the base shear strength of a structural system with given yield displacement and backbone capacity curve. Using publicly available software tools or closed-form solutions, YFS can be rapidly computed for any system that is satisfactorily approximated by a single-degree-of-freedom oscillator, e.g., as in any nonlinear static procedure application. Thus, stated performance objectives can be directly related to the strength and stiffness of the structure. The combination of ductility (or displacement) demand and mean annual frequency of exceedance that governs the design is readily determined, allowing either a code-compatible or code-exceeding design to be realized.

2. INTRODUCTION

Performance-based seismic design (PBSD) has received significant attention following large economic losses in the 1994 Northridge and 1995 Hyogo-Ken Nambu Earthquakes. Rather than focusing only on a life-safety performance level, PBSD targets multiple performance objectives, each typically defined as not exceeding a prescribed structural response level with a mean annual frequency higher than specified. One may also seek specific non-exceedance rates of economic losses or even casualties, echoing the definition of decision variables that are embedded in the Cornell-Krawinkler framework [1] of the Pacific Earthquake Engineering Research (PEER) Center.

Unfortunately, PBSD remains a difficult proposition. As the functional relationship between the design variables and the performance objectives is not invertible, considerable iteration is required. Each cycle encompasses re-design and re-analysis of the structure, where the latter is a full-blown performance-based assessment involving nonlinear static or dynamic runs. Any method built on this paradigm essentially becomes an iterated assessment procedure. Conceptual support for such a design paradigm is provided by Krawinkler et al. [2]. Researchers have also chosen to improve upon the efficiency of the re-design, often leading to the use of numerical optimization (Fragiadakis and Lagaros [3]).

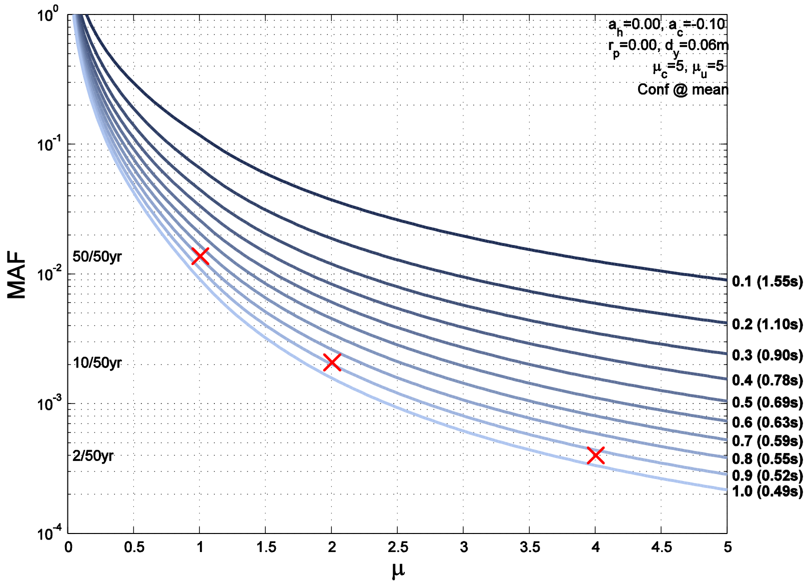


Fig. 1 YFS contours at C_y = 0.1,…,1.0 determined for an elastoplastic system (δ_y = 0.06m) at Van Nuys, CA, along with red "x" symbols that represent three performance objectives (μ = 1, 2, 4 at 50%, 10% and 2% in 50yrs exceedance rates, respectively). The third objective governs with $C_y \approx 0.93$. The corresponding period is $T \approx 0.51s$.

Despite the usefulness of current approaches, their implementation is not trivial. The link between a performance objective and the resulting design is obscure, coming out of numerous steps of numerical analysis. As an alternative, so-called "Yield Frequency Spectra" (YFS) are proposed as a design aid, being a direct visual representation of a system's performance that quantitatively links the mean annual frequency (MAF) of exceeding any displacement value (or ductility μ) with the system yield strength (or seismic coefficient C_y). YFS are plotted for a specified yield displacement; thus, periods of vibration represented in YFS vary with C_y. Fig. 1 presents an example for an elastic-perfectly-plastic oscillator. In this case, three performance objectives are specified (the red "x" symbols) while curves representing the site hazard convolved with the system fragility are plotted for fixed values of C_y. Of course, increases in C_y always reduce the MAF of exceeding a given ductility value. Thus, the minimum acceptable C_y (within some tolerance) that fulfils the set of performance objectives for the site hazard can be determined for a given single-degree-of-freedom system. This strength is used as a starting point for the PBSD of more complex structures. The performance-based design problem potentially can be solved in a single step with a good estimate of the yield displacement.

3. BASIS OF DESIGN

The design of a multi-degree-of-freedom structure will always involve some level of iteration. Thus, a truly direct performance-based design is probably unrealizable. To reduce the number of design/analysis cycles, we seek simplified models and stable parameters. One obvious shortcut, which actually forms the basis of all current seismic codes, is to rely on an SDOF system approximation. We will use this approximation for representing system level displacement (and ductility) responses. A second shortcut is to rely on the stability of the yield displacement—the notion that the yield drift ratio of a bilinear approximation to the first mode pushover curve is stable with changes in strength. The changes in strength affect stiffness and drift (or ductility) demands.

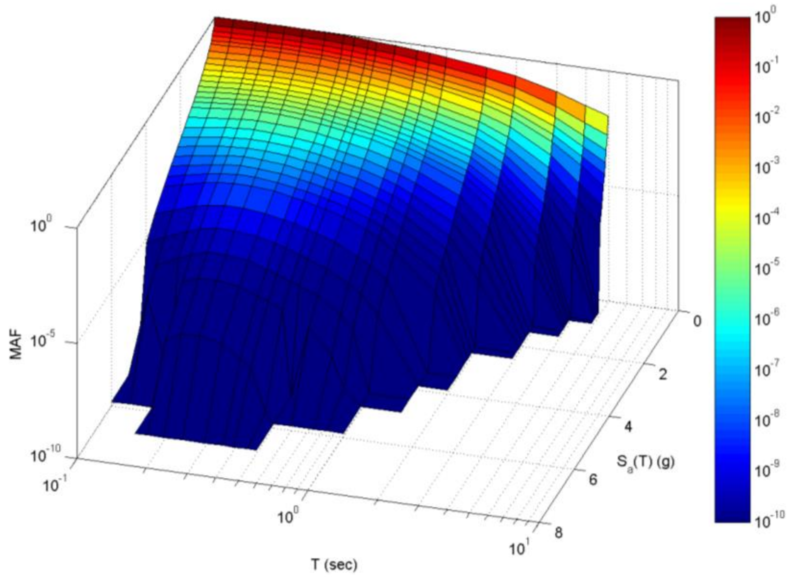
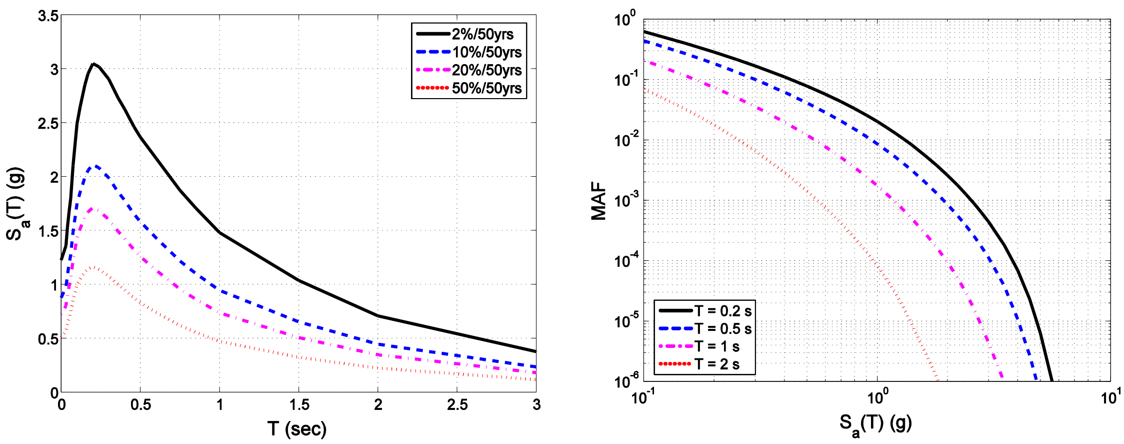


Fig. 2 Spectral acceleration hazard surface for Van Nuys, CA.



(a) (b)

Fig. 3 (a) Uniform hazard spectra and (b) S_a hazard curves for Van Nuys, CA.

The essential ingredients of our approach to PBSD are (a) the site hazard and (b) some assumption about the system's behavior (e.g. elastic, elastoplastic etc). Comprehensive site hazard representation that is compatible with current design norms can be achieved by the seismic hazard surface, a 3D plot of the MAF of exceeding any level of spectral acceleration for the full practical range of periods (Fig. 2). This is the true representation of the seismic loads for any given site. More familiar pictures can be produced from the hazard surface by taking cross-section (or contours). Cutting horizontally at given values of MAF will provide the corresponding uniform hazard spectra (UHS). For example, at P_o = $-\ln(1\text{-}0.1)/50 = 0.0021$, or a 10% in 50yrs probability of exceedance (Fig. 3a), one gets the spectrum typically associated with design at the ultimate limit-state (or Life Safety). A cross-section at a given period T produces the corresponding $S_a(T)$ hazard curve (Fig. 3b).

For a given capacity curve shape (or system type) the design target is to estimate the yield strength and the period T for not exceeding a limiting displacement δ_{lim} at a rate higher than P_o. Even for an SDOF system, the introduction of yielding, ductility and the resulting record-to-record response variability make this a challenging problem. This is best represented in the familiar coordinates of intensity measure (IM), here being the first mode spectral acceleration $S_a(T)$, and engineering demand parameter (EDP), i.e., the displacement response δ. The structural response then appears in the form of incremental dynamic analysis (IDA, Vamvatsikos and Cornell [4]) curves as shown in Fig. 4 for a $T = 1$s system with a capacity curve having positive and then a negative post-yield stiffness. Cornell et al [5] have shown that additional hazard levels beyond P_o need to be considered in evaluating the system's performance due to response variability. The reason is that values lower than the average response for the seismic intensity corresponding to P_o appear more frequently (i.e., correspond to a higher hazard rate in Fig. 3b). Hence, they tend to contribute significantly more to the system's rate of exceeding $\delta = \delta_{lim}$. Formally, this relationship may be represented by the following integral [6]:

$$\lambda(\delta) = \int_0^{+\infty} F\left(S_{ac}(\delta) \,|\, s\right) \left|\mathrm{d}H(s)\right| \tag{1}$$

where $\lambda(\cdot)$ is the MAF of exceeding δ. $S_{ac}(\delta)$ is the (random) limit-state capacity, representing the minimum intensity level for a ground motion record to cause exceedance of displacement δ (e.g., Fig. 4**)**. $F(\cdot)$ is the cumulative distribution function (CDF) of S_{ac} evaluated at a spectral acceleration value of s, and $H(s)$ is the associated hazard rate. The absolute value is needed for the differential of $H(s)$ because the hazard is monotonically decreasing, thus always having a negative slope.

The seismic code foregoes such considerations through implicit incorporation of two assumptions: (a) Using the strength reduction R or behavior factor q to account for the effect of yielding and ductility in the mean/median response, (b) ignoring the effect of dispersion, assuming that the seismic loads consistent with P_o are enough to guarantee a similar (or lower) rate of non-exceedance of δ_{lim}. The error due to the above is "covered" by employing various implicit conservative approximations to account for the effect of the previous non-conservative assumptions, typically through the selection of R (or q).

The magnitude of the assumptions is such that one can never be sure of achieving the stated objective(s). The margin of safety depends on the site and the system. Even when safe, the design is typically far from optimal. Essentially, we lack information on where exactly a design resides on this wide margin between meeting and failing the performance criteria. Even worse, as any calibration for safety has been performed on the basis of the standard code assumptions of what an acceptable performance is, it is not possible to accurately inject one's own stricter criteria for a better performing structure. The importance factors used to amplify the design spectrum are only a poor substitute.

As a solution we aim to offer a practical and theoretically consistent procedure that can fully resolve the inelastic SDOF design problem. This will be built upon (a) Eq. 1 for estimating structural performance, (b) the R-μ-T relationships for estimating the probabilistic distribution of structural response given intensity and (c) a yield displacement basis for design, by virtue of being a far more stable system parameter compared to the period [7,8]. In a graphical format, this solution is represented via YFS.

4. ORIGIN, DEFINITION, AND USE OF YFS

For a yielding system, the direct equivalent of elastic spectral acceleration or spectral displacement hazard curves are the inelastic displacement (or drift) hazard curves [9, 10]. These may be determined by using Eq. 1 to estimate the MAF of exceeding any limiting value of displacement [6]. While useful for assessment, they lack the necessary parameterization for design. An appropriate normalization may be achieved for oscillators with yield strength and displacement of F_y and δ_y, respectively, by employing ductility μ, rather than displacement δ, and the seismic coefficient C_y instead of the strength:

$$\mu = \frac{\delta}{\delta_y}, \qquad C_y = \frac{F_y}{W} \tag{2}$$

where W is the weight and F_y the yield base shear. For SDOF systems C_y is numerically equivalent to $S_{ay}(T,\xi)$ / g, i.e. the spectral acceleration value to cause yield in units of g, at the period T and viscous damping ratio ξ of the system.

Up to this point, what has been proposed is not fundamentally different from the results presented by Ruiz-Garcia and Miranda [11] on the derivation of maximum inelastic displacement hazard curves. What makes the difference is defining δ_y as a constant for a given structural system [7,8]. Then, C_y becomes a direct replacement of the period T:

$$T = 2\pi\sqrt{\frac{\delta_y}{C_y g}}, \quad \text{or} \qquad C_y = \frac{\delta_y}{g}\left(\frac{2\pi}{\mathrm{T}}\right)^2 \tag{3}$$

For a given site hazard, system damping, δ_y, value of C_y (or period), and capacity curve *shape* (e.g. as normalized in terms of $R = F/F_y$ and μ), a unique representation of the system's probabilistic response may be gained through the displacement (or ductility) hazard curves produced via Eq. 1. Damping, δ_y and the capacity curve shape are considered as stable system characteristics. By plotting such curves of $\lambda(\mu)$, for a range of μ and C_y values and, we can get contours of the inelastic displacement hazard surface for constant values of C_y. These contours allow the direct evaluation of system strength and period—i.e., the C_y required to satisfy any combination of performance objectives defined as $P_o = \lambda(\mu_{lim})$, where each limiting value of ductility μ_{lim} is associated with a maximum MAF of exceedance P_o, as shown in Fig. 1. The practical estimation of YFS is thus based on the case-by-case solution of Eq. 1. This involves a comprehensive evaluation for a number of SDOF oscillators with the same capacity curve shape and yield displacement but different periods and yield strengths. If a numerical approach is employed, then we can obtain the YFS shown in Fig. 1 at the cost of a few minutes of computer time.

5. EXAMPLE APPLICATION

For showcasing the methodology, a 4-story steel moment resisting frame will be designed for a site in Van Nuys, CA (Fig. 2). It has uniform story height of 3.6m, total height of H = 14.4m and L = 9m beam spans. The interstory drift limit for Damage Limitation (DL) is θ_{lim} = 0.75% and the required ductility is 3.0 for the Strength Limitation (SL) checking. The allowable exceedance probabilities are 50% and 10% in 50yrs, respectively. Equal interstory drifts are assumed to occur throughout the height of the structure, at least in the elastic region. According to Aschheim [10], a simple way to calculate the yield roof drift (or any story yield drift) of a regular steel moment resisting frame is

$$\theta_y = \frac{\varepsilon_y}{6}\left(\frac{h}{d_{col}COF} + \frac{2L}{d_{bm}}\right) \tag{4}$$

where ε_y is the yield strain of steel, h the story height, L the beam span, COF the column overstrength factor and d_{col}, d_{bm} the column and beam depth, respectively. Let ε_y = 0.18% (for f_y = 355MPa steel), h = 3.6m, L = 9m, COF = 1.3 (suggested values are 1.2 – 1.5), d_{col} = 0.6m, d_{bm} = 0.70m. Then, θ_y = 0.9%, and the limiting ductility for SLS becomes μ_{limSLS} = 0.84. For a typical first-mode participation factor Γ = 1.3, the equivalent SDOF yield displacement is $\delta_y = \theta_y H/\Gamma$ = 0.10m

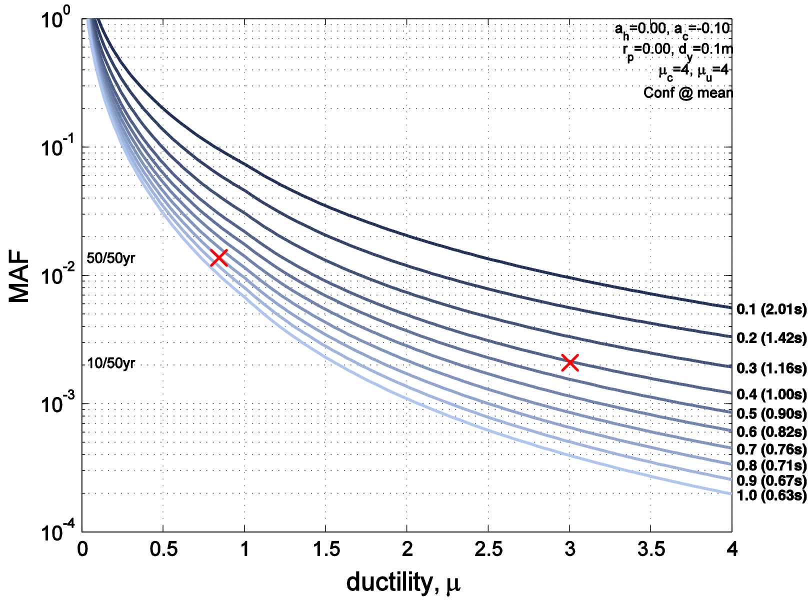


Fig. 5 YFS contours at C_y = 0.1,…,1.0 for designing a 4-story steel frame at Van Nuys, CA. The red "x" symbols represent two performance objectives (μ = 0.84, 3 at 50% and 10% in 50yrs exceedance rates, respectively). The first objective governs with $C_y \approx 0.81$ corresponding to a period of $T \approx 0.71$s.

Let the dispersions due to epistemic uncertainty be 20% and 30% for DL and SL, respectively and assume that the system response is roughly elastoplastic. As expected for a moment-resisting steel frame, DL governs. By employing the estimated YFS of Fig. 5 (for a confidence level consistent with the mean MAF estimate) the result is C_y = 0.81 corresponding to a period of T = 0.71sec. At this point, we can consider the beneficial effects of overstrength and further reduce C_y. For example, by employing a conservative value of, say, 1.50, the suggested seismic coefficient would become 0.54. This value can now be applied to prescribe the seismic loads, e.g., in a code-compatible setting. The end result may not be perfect, but it is close to fully satisfying the stated objectives, something that is not as straightforward when using just a design spectrum as the point of entry.

6. CONCLUSIONS

Yield Frequency Spectra have been introduced as an intuitive and practical approach to perform approximate performance-based design. They are a simple tool for considering an arbitrary number of objectives that can be connected to the global displacement of an equivalent single-degree-of-freedom oscillator. For this relatively benign limitation, our approach can help deliver preliminary designs that are close to their performance targets, requiring only limited re-analysis and re-design cycles to reach the final stage.

7. ACKNOWLEDGEMENTS

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ΑΜΕΣΟΣ ΣΕΙΣΜΙΚΟΣ ΣΧΕΔΙΑΣΜΟΣ ΒΑΣΕΙ ΕΠΙΤΕΛΕΣΤΙΚΟΤΗΤΑΣ ΕΝΟΣ ΜΕΤΑΛΛΙΚΟΥ ΠΛΑΙΣΙΑΚΟΥ ΦΟΡΕΑ ΜΕ ΤΗ ΧΡΗΣΗ ΦΑΣΜΑΤΩΝ ΣΥΧΝΟΤΗΤΑΣ ΔΙΑΡΡΟΗΣ

Δημήτριος Βαμβάτσικος
Σχολή Πολιτικών Μηχανικών
Εθνικό Μετσόβιο Πολυτεχνείο, Αθήνα
e-mail: divamva@mail.ntua.gr

Μαρκ Α. Άσαϊμ
Τμήμα Πολιτικών Μηχανικών
Πανεπιστήμιο Santa Clara, Καλιφόρνια, ΗΠΑ
e-mail: maschheim@scu.edu

ΠΕΡΙΛΗΨΗ

Τα Φάσματα Διαρροής-Συχνότητας (ΦΔΣ) χρησιμοποιούνται για τον άμεσο σεισμικό σχεδιασμό ενός τετραώροφου μεταλλικού πλασιακού φορέα για δεδομένους στόχους επιτελεστικότητας. Τα ΦΔΣ προσφέρουν μια μοναδική οπτική του χώρου των δυνατών λύσεων για την επιτελεστικότητα των κατασκευών, όπως αυτή εκφράζεται σε όρους μέσης ετήσιας συχνότητας (ΜΕΣ) υπέρβασης ορίων πλαστιμότητας (ή μετακίνησης) έναντι της αντοχής σε τέμνουσα βάσης ενός δομικού συστήματος με δεδομένη μετακίνηση διαρροής και καμπύλη ικανότητας. Χρησιμοποιώντας ελεύθερο λογισμικό ή κλειστές αναλυτικές λύσεις, τα ΦΔΣ μπορούν να υπολογιστούν τάχιστα για οποιοδήποτε σύστημα που μπορεί να προσομοιωθεί ικανοποιητικά από ένα μονοβάθμιο ταλαντωτή, κατά τα πρότυπα της ανελαστικής στατικής ανάλυσης. Με αυτόν τον τρόπο δεδομένοι στόχοι επιτελεστικότητας μπορούν να συσχετιστούν απευθείας με την αντοχή και τη δυσκαμψία της κατασκευής. Υπολογίζεται δε εύκολα ο συνδυασμός της πλαστιμότητας (ή μετακίνησης) και της ΜΕΣ υπέρβασης που κυριαρχεί στη διαστασιολόγηση, επιτρέποντας έτσι ένα σχεδιασμό που μπορεί να είναι συμβατός με τον αντισεισμικό κώδικα ή, κατά βούληση, να τον υπερβαίνει.