GENERALIZED WARPING IN PRE- AND POST-BUCKLING ANALYSIS OF COMPOSITE BEAMS

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Keywords: Nonuniform warping, Shear lag, Flexure, Torsion, Shear deformation, Composite beams, Flexural-torsional buckling, Postbuckling analysis, Non-linear analysis

ABSTRACT

Shear lag phenomenon has been observed in many structural members (e.g. beams of box-shaped cross sections, folded structural members). Its influence is considered to be much more important in non-linear analysis. However neither Euler – Bernoulli (cross sections remain plane and normal to the deformed axis) nor Timoshenko (cross sections remain plane but not normal to the deformed axis) beam theory is able to take into account non-uniform warping, that is related with shear lag phenomenon, because both of them maintain the assumption that plane cross sections remain plane after deformation. In order to avoid the usage of 2d or 3d theory of elasticity models, in this paper, a beam finite element is employed for the postbuckling analysis of arbitrarily loaded and shaped, composite cross section, taking into account generalized warping, i.e. shear lag due to both flexure and torsion. The nonuniform warping distributions are taken into account by using four independent warping parameters multiplying a shear warping function in each direction and two torsional warping functions, respectively, which are obtained by solving the corresponding boundary value problems, formulated exploiting the longitudinal local equilibrium equation. A shear stress correction is also performed to improve the stress field arising from the employed kinematical considerations. The finite element equations are formulated with respect to the independent warping parameters additionally to the displacement and rotation components. The influence of shear lag phenomenon in postbuckling analysis is investigated through a numerical example with practical interest.

1. INTRODUCTION

In most cases in the analysis of beam-like structures, Euler – Bernoulli beam theory assumptions are adopted, while in the case of non-negligible shear deformation effect, these assumptions are relaxed by using Timoshenko beam theory. However, both theories maintain the assumption that plane cross sections remain plane (no out-of-plane deformation). In order to take into account shear lag effects in the context of a beam theory, the inclusion of non-uniform warping is necessary, relaxing the assumption of plane cross section. For this purpose the so-called higher order beam theories have been developed taking into account shear lag [1] effects.

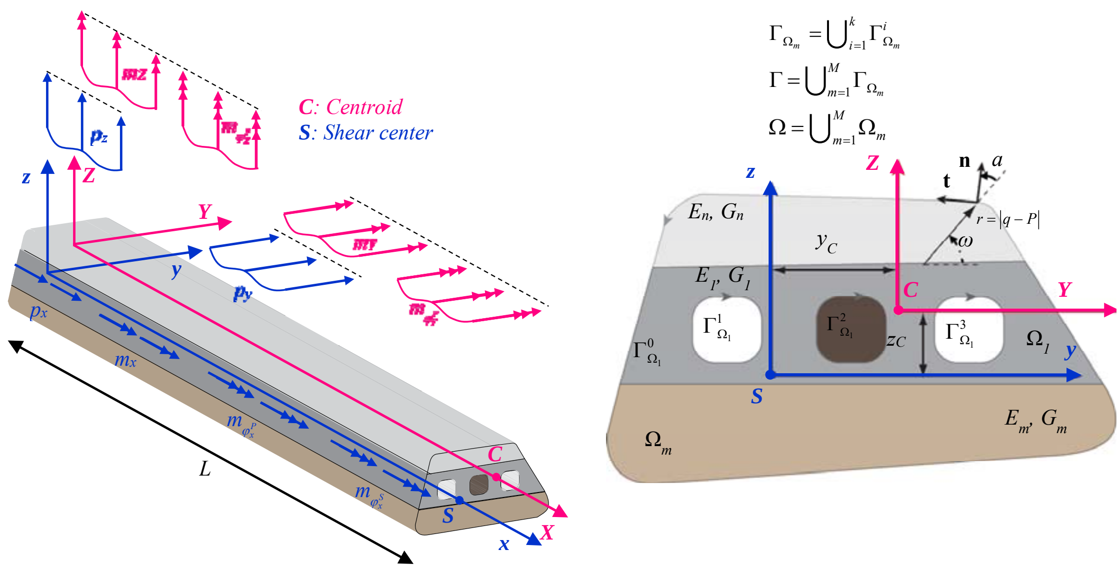
The essential features and novel aspects of the proposed formulation compared with previous ones are summarized as follows.

i) It takes into account generalized warping (warping and shear lag due to shear, flexure and torsion) in buckling analysis of arbitrarily shaped, prismatic, composite beams.
ii) It performs buckling analysis based on a higher-order beam theory that is of increased interest due to its important advantages over refined approaches such as 3-D solutions.
iii) The beam is supported by the most general boundary conditions including elastic support or restraint.
iv) The influence of shear lag phenomenon [1] in postbuckling analysis [2] is investigated through a numerical example with practical interest.

2. STATEMENT OF THE PROBLEM

2.1. Displacement, strain and stress components

Let us consider a prismatic beam of length L (Fig. 1), of constant arbitrary cross-section A . The cross section consists of materials in contact, each of which can surround a finite number of inclusions, with modulus of elasticity E_m and shear modulus G_m, occupying the regions Ω_m (*m=1,2,...,M*) of the y,z plane (Fig. 1b). The materials of these regions are assumed homogeneous, isotropic and linearly elastic i.e. the term composite is not referred to layered beams. Let also the boundaries of the nonintersecting regions Ω_m be denoted by Γ_m $\left(m=1,2,...,M\right)$. These boundary curves are piecewise smooth, i.e. they may have a finite number of corners. The reference material of the cross section has modulus of elasticity E and shear modulus G. The geometric characteristics of the composite cross section are calculated as described in [1]. In Fig. 1b CYZ is the coordinate system through the cross section's centroid C, while y_C, z_C are its coordinates with respect to Syz principal shear system of axes through the cross section's shear center S. The beam can be supported by the most general linear boundary conditions and is subjected to the combined action of the arbitrarily distributed or concentrated axial loading $p_x(X)$ along X direction, transverse loading $p_y(x)$ and $p_z(x)$ along the y, z directions, respectively, twisting moment $m_x(x)$ along x direction, bending moments $m_Y(x)$, $m_Z(x)$ along Y, Z directions, respectively, as well as bending $m_{\varphi_Y^P}(x)$, $m_{\varphi_Z^P}(x)$ and primary and secondary torsional $m_{\varphi_x^P}(x)$, $m_{\varphi_x^S}(x)$ warping moments.



(a) (b)

Fig. 1. Prismatic beam under loading (a) with a composite cross section of arbitrary shape occupying the two dimensional region Ω *(b).*

Under the aforementioned loading the displacement field of the beam with respect to the Syz system of axes is given as

$$\begin{aligned}\bar{u}(x,y,z) &= \bar{u}^P(x,y,z)+\bar{u}^S(x,y,z)= \\ &= \underbrace{u(x)+\bar{\theta}_Y(x)Z-\bar{\theta}_Z(x)Y+\eta_x(x)\phi_x^P(y,z)+}_{\text{primary}} \\ &\quad \underbrace{+\eta_Y(x)\phi_Y^P(y,z)+\eta_Z(x)\phi_Z^P(y,z)+\xi_x(x)\phi_x^S(y,z)}_{\text{secondary}} \end{aligned} \tag{1a}$$

$$\bar{v}(x,y,z)=v(x)-z\cdot\sin(\theta_x(x))-y\cdot(1-\cos(\theta_x(x))) \tag{1b}$$

$$\bar{w}(x,y,z)=w(x)+y\cdot\sin(\theta_x(x))-z\cdot(1-\cos(\theta_x(x))) \tag{1c}$$

$$\bar{\theta}_Y(x)=\theta_Z(x)\cdot\sin(\theta_x(x))+\theta_Y(x)\cdot\cos(\theta_x(x)) \tag{1d}$$

$$\bar{\theta}_Z(x)=\theta_Z(x)\cdot\cos(\theta_x(x))-\theta_Y(x)\cdot\sin(\theta_x(x)) \tag{1e}$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are the axial and transverse beam displacement components with respect to the $Sxyz$ system of axes, while $\bar{u}^P$, $\bar{u}^S$ denote the primary and secondary longitudinal displacements, respectively. Moreover, $v(x)$, $w(x)$ describe the deflection of the center of twist S, while $u(x)$ denotes the "average" axial displacement of the cross section. $\theta_Z(x)$, $\theta_Y(x)$ are the angles of rotation due to bending about the centroidal Z, Y axes, respectively; $\eta_x(x)$, $\xi_x(x)$ are the independent warping parameters introduced to describe the nonuniform distribution of primary and secondary torsional warping, while $\eta_Y(x)$, $\eta_Z(x)$ are the independent warping parameters introduced to describe the nonuniform distribution of primary warping due to shear. $\varphi_x^P(y,z)$, $\varphi_x^S(y,z)$ are the primary and secondary torsional warping functions with respect to the center of twist S [3] while

$\varphi_Y^P(y,z)$, $\varphi_Z^P(y,z)$ are the primary shear warping functions with respect to the centroid C. It holds that $Z = z - z_C$, $Y = y - y_C$.

Substituting Eqs.(1a)-(1c) in the non-linear (Green) strain-displacement relations of the non-vanishing strains

$$\varepsilon_{xx} = \frac{\partial \bar{u}}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial \bar{u}}{\partial x}\right)^2 + \left(\frac{\partial \bar{v}}{\partial x}\right)^2 + \left(\frac{\partial \bar{w}}{\partial x}\right)^2\right] \qquad (2a)$$

$$\gamma_{xy} = \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y}\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{w}}{\partial y}\frac{\partial \bar{w}}{\partial x} \qquad \text{(REF \_Ref443840057 \textbackslash h 2b)}$$

$$\gamma_{xz} = \frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{u}}{\partial z}\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial z}\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{w}}{\partial z}\frac{\partial \bar{w}}{\partial x} \qquad \text{(REF \_Ref443840057 \textbackslash h 2c)}$$

($\left(\frac{\partial \bar{u}}{\partial x}\right)^2 << \frac{\partial \bar{u}}{\partial x}$, $\left(\frac{\partial \bar{u}}{\partial x}\right)\left(\frac{\partial \bar{u}}{\partial z}\right) << \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial z}$, $\left(\frac{\partial \bar{u}}{\partial x}\right)\left(\frac{\partial \bar{u}}{\partial y}\right) << \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial y}$, i.e. the terms $\left(\frac{\partial \bar{u}}{\partial x}\right)\left(\frac{\partial \bar{u}}{\partial z}\right)$ and $\left(\frac{\partial \bar{u}}{\partial x}\right)\left(\frac{\partial \bar{u}}{\partial y}\right)$ have been neglected as considered to be small compared with the linear and the rest of the non-linear terms) the non-vanishing strain resultants are given as

$$\begin{aligned}
\varepsilon_{xx} = {} & \overbrace{\underbrace{u_{,x} + \theta_{Y,x}Z - \theta_{Z,x}Y + n_{x,x}\varphi_x^P}_{\text{primary}} + \underbrace{n_{Y,x}\varphi_Y^P + n_{Z,x}\varphi_Z^P + \xi_{,x}\varphi_x^S}_{\text{secondary}}}^{\text{linear part}} \\
& \left.\begin{aligned}
&+ \left(\bar{\theta}_{Y,x} - \theta_{Y,x}\right)Z - \left(\bar{\theta}_{Z,x} - \theta_{Z,x}\right)Y + \frac{1}{2}\left(v_{,x}^{\;2} + w_{,x}^{\;2}\right) + \frac{1}{2}\theta_{x,x}^{\;2}\left(y^2 + z^2\right) \\
&+ \theta_{x,x}\left(-v_{,x}\left(z\cos(\theta_x) + y\sin(\theta_x)\right) + w_{,x}\left(y\cos(\theta_x) - z\sin(\theta_x)\right)\right)
\end{aligned}\right\} \text{non-linear part}
\end{aligned} \qquad (3a)$$

$$\begin{aligned}
\gamma_{xy} = {} & \left.\begin{aligned}
&\underbrace{\gamma_Z^P\left(Z_{,y} + \varphi_{Y,y}^P\right) + \gamma_Y^P\left(Y_{,y} + \varphi_{Z,y}^P\right) + \gamma_x^P\left(-z + \varphi_{x,y}^P\right)}_{\text{primary}} + \\
&+ \underbrace{\gamma_Z^S\varphi_{Y,y}^P + \gamma_Y^S\varphi_{Z,y}^P + \gamma_x^S\left(\varphi_{x,y}^P + \varphi_{x,y}^S\right)}_{\text{secondary}} + \underbrace{\gamma_x^T\varphi_{x,y}^S}_{\text{tertiary}}
\end{aligned}\right\} \text{linear part} \\
& \underbrace{+\left(\theta_Y + w_{,x}\right)\sin(\theta_x) + \left(\theta_Z - v_{,x}\right)\left(1 - \cos(\theta_x)\right)}_{\text{non-linear part}}
\end{aligned} \qquad (3b)$$

$$\begin{aligned}
\gamma_{xz} = {} & \left.\begin{aligned}
&\underbrace{\gamma_Z^P\left(Z_{,z} + \varphi_{Y,z}^P\right) + \gamma_Y^P\left(Y_{,z} + \varphi_{Z,z}^P\right) + \gamma_x^P\left(y + \varphi_{x,z}^P\right)}_{\text{primary}} + \\
&+ \underbrace{\gamma_Z^S\varphi_{Y,z}^P + \gamma_Y^S\varphi_{Z,z}^P + \gamma_x^S\left(\varphi_{x,z}^P + \varphi_{x,z}^S\right)}_{\text{secondary}} + \underbrace{\gamma_x^T\varphi_{x,z}^S}_{\text{tertiary}}
\end{aligned}\right\} \text{linear part} \\
& \underbrace{+\left(\theta_Z - v_{,x}\right)\sin(\theta_x) - \left(\theta_Y + w_{,x}\right)\left(1 - \cos(\theta_x)\right)}_{\text{non-linear part}}
\end{aligned} \qquad (3c)$$

or

$$\varepsilon_{xx} = \varepsilon_{xx}^{lin} + \varepsilon_{xx}^{nl} \tag{4a}$$

$$\gamma_{xy} = \gamma_{xy}^{lin} + \gamma_{xy}^{nl} \tag{4b}$$

$$\gamma_{xz} = \gamma_{xz}^{lin} + \gamma_{xz}^{nl} \tag{4c}$$

where, $(\cdot)_{,i}$ denotes differentiation with respect to i.

Considering strains to be small, employing the second Piola-Kirchhoff stress tensor, the stress components in the region Ω_m $(m = 1,2,...,M)$ are defined in terms of the strain ones as

$$\begin{Bmatrix} (S_{xx})_m \\ (S_{xy})_m \\ (S_{xz})_m \end{Bmatrix} = \begin{bmatrix} E_m^* & 0 & 0 \\ 0 & G_m & 0 \\ 0 & 0 & G_m \end{bmatrix} \begin{Bmatrix} (\varepsilon_{xx})_m \\ (\gamma_{xy})_m \\ (\gamma_{xz})_m \end{Bmatrix} \tag{5}$$

where E_m^* is obtained from Hooke's stress-strain law as $E_m^* = E_m(1-\nu_m)/\left[(1+\nu_m)(1-2\nu_m)\right]$. In this study E_m is considered instead of E_m^* ($E_m^* \approx E_m$). Employing Eqs. (3a)-(3c) and after the stress correction for the linear part of shear stresses $\left(S_{xy}^{lin}\right)_m, \left(S_{xz}^{lin}\right)_m$ that is presented in [1]

$$\begin{aligned}
(S_{xx})_m = {} & \overbrace{\underbrace{E_m\left(u_{,x} + \theta_{Y,x}Z - \theta_{Z,x}Y + n_{x,x}\left(\varphi_x^P\right)_m\right)}_{\text{primary}} + \underbrace{E_m\left(n_{Y,x}\left(\varphi_Y^P\right)_m + n_{Z,x}\left(\varphi_Z^P\right)_m + \xi_{,x}\left(\varphi_x^S\right)_m\right)}_{\text{secondary}}}^{\text{linear part}} \\
& \left.\begin{aligned} & + E_m\left(\left(\bar{\theta}_{Y,x} - \theta_{Y,x}\right)Z - \left(\bar{\theta}_{Z,x} - \theta_{Z,x}\right)Y + \frac{1}{2}\left(v_{,x}{}^2 + w_{,x}{}^2\right) + \frac{1}{2}\theta_{x,x}{}^2\left(y^2 + z^2\right)\right) \\ & + E_m\,\theta_{x,x}\left(-v_{,x}\left(z\cos(\theta_x) + y\sin(\theta_x)\right) + w_{,x}\left(y\cos(\theta_x) - z\sin(\theta_x)\right)\right) \end{aligned}\right\} \text{non-linear part}
\end{aligned} \tag{6a}$$

$$\begin{aligned}
(S_{xy})_m = {} & \left.G_m\left(\underbrace{\gamma_Z^P\left(\Phi_{Y,y}^P\right)_m + \gamma_Y^P\left(\Phi_{Z,y}^P\right)_m + \gamma_x^P\left(-z + \left(\varphi_{x,y}^P\right)_m\right)}_{\text{primary}} + \underbrace{\gamma_Z^S\left(\Phi_{Y,y}^S\right)_m + \gamma_Y^S\left(\Phi_{Z,y}^S\right)_m + \gamma_x^S\left(\Phi_{x,y}^S\right)_m}_{\text{secondary}} + \underbrace{\gamma_x^T\left(\Phi_{x,y}^T\right)_m}_{\text{tertiary}}\right)\right\} \text{linear part} \\
& + \underbrace{G_m\left(\left(\theta_Y + w_{,x}\right)\sin(\theta_x) + \left(\theta_Z - v_{,x}\right)\left(1 - \cos(\theta_x)\right)\right)}_{\text{non-linear part}}
\end{aligned} \tag{6b}$$

$$\left(S_{xz}\right)_m = G_m \left(\left. \begin{aligned} &\underbrace{\gamma_Z^P \left(\Phi_{Y,z}^P\right)_m + \gamma_Y^P \left(\Phi_{Z,z}^P\right)_m + \gamma_x^P \left(y + \left(\varphi_{x,z}^P\right)_m\right)}_{\text{primary}} \\ &+ \underbrace{\gamma_Z^S \left(\Phi_{Y,z}^S\right)_m + \gamma_Y^S \left(\Phi_{Z,z}^S\right)_m + \gamma_x^S \left(\Phi_{x,z}^S\right)_m}_{\text{secondary}} + \underbrace{\gamma_x^T \left(\Phi_{x,z}^T\right)_m}_{\text{tertiary}} \end{aligned} \right\} \text{linear part} \right) \\ + \underbrace{G_m \left(\left(\theta_Z - v_{,x}\right) \sin\left(\theta_x\right) - \left(\theta_Y + w_{,x}\right)\left(1 - \cos\left(\theta_x\right)\right)\right)}_{\text{non-linear part}} \tag{6c}$$

or

$$\left(S_{xx}\right)_m = \left(S_{xx}^{lin}\right)_m + \left(S_{xx}^{nl}\right)_m \tag{7a}$$

$$\left(S_{xy}\right)_m = \left(S_{xy}^{lin}\right)_m + \left(S_{xy}^{nl}\right)_m \tag{7b}$$

$$\left(S_{xz}\right)_m = \left(S_{xz}^{lin}\right)_m + \left(S_{xz}^{nl}\right)_m \tag{7c}$$

where $\gamma_Y^P = v_{,x} - \theta_Z$, $\gamma_Y^S = \eta_Z - v_{,x} + \theta_Z$, $\gamma_Z^P = w_{,x} + \theta_Y$, $\gamma_Z^S = \eta_Y - w_{,x} - \theta_Y$, $\gamma_x^P = \theta_{x,x}$, $\gamma_x^S = \eta_x - \theta_{x,x}$ and $\gamma_x^T = \xi_x - \eta_x + \theta_{x,x}$ are "average" shear strain quantities, while

$$\Phi_x^S = \varphi_x^P + \varphi_x^S \qquad \Phi_x^T = \varphi_x^S + \varphi_x^T \tag{8a,b}$$

$$\Phi_Y^P = Z + \varphi_Y^P \qquad \Phi_Y^S = \varphi_Y^P + \varphi_Y^S \tag{8c,d}$$

$$\Phi_Z^P = Y + \varphi_Z^P \qquad \Phi_Z^S = \varphi_Z^P + \varphi_Z^S \tag{8e,f}$$

2.2. Principle of virtual work

According to principle of virtual work

$$\delta U = \delta W \tag{9}$$

where δU and δW are the virtual work of the internal and external actions of the beam respectively.

2.3. Warping functions

The warping functions φ_x^P, Φ_Y^P, Φ_Z^P, Φ_x^S, Φ_Y^S, Φ_Z^S, Φ_x^T that have been employed in section 2.1 are obtained after solving corresponding boundary value problems, formulated exploiting the local equilibrium equations in the longitudinal direction and the corresponding boundary conditions as presented in [1].

3. NUMERICAL SOLUTION

The described problem is numerically solved employing the Finite Element Method (FEM) for the discretization and the incremental solution of Eq. (9).

4. NUMERICAL EXAMPLE AND CONCLUSIONS

In this example a cantilever beam of length L=1m consisting of a IPE300, steel ($E = 2.1 \cdot 10^8\, kN/m^2$, $v = 0.3$) cross section is analyzed. The free end of the beam is subjected to a concentrated load along its weak axis. The resulted load – displacement curves are presented in Fig. 2 as calculated by Generalized Warping Beam Theory (GWBT - present study), Euler – Bernoulli Beam Theory including (E/BBT+warp) or not (E/BBT) primary torsional warping, using a code of the authors, and 3-D solid Finite Elements using a commercial FEM package with solid modelling capabilities [4]. The importance of shear lag phenomenon is highlighted through the results of the treated example.

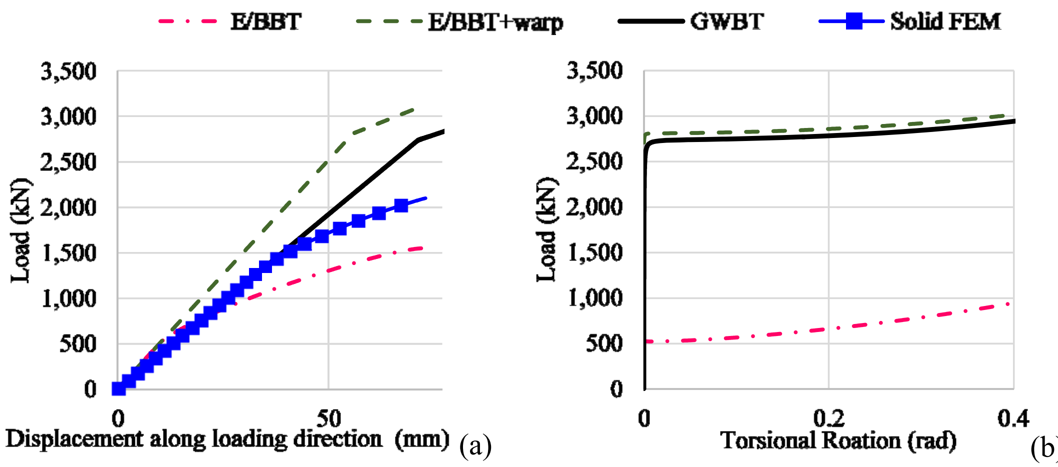


Fig. 2. Load with respect to displacement along loading direction (a) and torsional rotation (b) of the beam of the numerical example.

ACKNOWLEDGEMENTS

This research work was supported by "Onassis Foundation", "Foundation for Education and European Culture" and "A.G. Leventis Foundation".

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ΓΕΝΙΚΕΥΜΕΝΗ ΣΤΡΕΒΛΩΣΗ ΣΤΗΝ ΠΡΟ- ΚΑΙ ΜΕΤΑ-ΛΥΓΙΣΜΙΚΗ ΑΝΑΛΥΣΗ ΣΥΜΜΙΚΤΩΝ ΔΟΚΩΝ

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Keywords: Ανομοιόμορφη στρέβλωση, Διατμητική υστέρηση, Κάμψη, Στρέψη, Διατμητική παραμόρφωση, Σύμμικτες δοκοί, Στρεπτοκαμπτικός λυγισμός, Μεταλυγισμική ανάλυση, Μη γραμμική ανάλυση

ΠΕΡΙΛΗΨΗ

Το φαινόμενο της διατμητικής υστέρησης έχει παρατηρηθεί σε πολλά δομικά μέλη (π.χ. δοκοί κυβωτιοειδούς διατομής, πτυχωτοί φορείς). Η επιρροή του θεωρείται πολύ πιο σημαντική στη μη γραμμική ανάλυση. Ωστόσο ούτε η θεωρία Euler –Bernoulli (η διατομή παραμένει επίπεδη και κάθετη στον παραμορφωμένο άξονα) ούτε η θεωρία Timoshenko (η διατομή παραμένει επίπεδη αλλά όχι κάθετη στον παραμορφωμένο άξονα) είναι ικανή να λάβει υπόψη την ανομοιόμορφη στρέβλωση, που σχετίζεται με το φαινόμενο της διατμητικής υστέρησης, επειδή και οι δύο θεωρίες διατηρούν την υπόθεση ότι επίπεδες διατομές παραμένουν επίπεδες μετά την παραμόρφωση. Προκειμένου να αποφευχθεί η χρήση μοντέλων 2-διάστατης ή 3-διάστατης ελαστικότητας, σε αυτή την εργασία, υλοποιείται ένα πεπερασμένο στοιχείο δοκού για τη μεταλυγισμική ανάλυσή δοκών υπό τυχούσα φόρτιση, τυχούσας σύμμικτης διατομής, λαμβάνοντας υπόψη τη γενικευμένη στρέβλωση, δηλαδή διατμητική υστέρηση εξ' αιτίας κάμψης και στρέψης. Η ανομοιόμορφη στρέβλωση λαμβάνεται υπόψη χρησιμοποιώντας τέσσερεις ανεξάρτητες παραμέτρους στρέβλωσης πολλαπλασιαζόμενες με δύο διατμητικές συναρτήσεις στρέβλωσης για κάμψη σε κάθε διεύθυνση και δύο στρεπτικές συναρτήσεις στρέβλωσης, αντίστοιχα, οι οποίες προκύπτουν επιλύοντας τα αντίστοιχα προβλήματα συνοριακών τιμών, που μορφώνονται από την εφαρμογή της διαμήκους τοπικής εξίσωσης ισορροπίας. Μια διόρθωση διατμητικών τάσεων υλοποιείται επίσης προκειμένου να βελτιώσει το πεδίο τάσεων που προκύπτει από τις κινηματικές θεωρήσεις που εφαρμόζονται. Οι εξισώσεις της μεθόδου πεπερασμένων στοιχείων μορφώνονται ως προς τις ανεξάρτητες παραμέτρους στρέβλωσης επιπρόσθετα στις συνιστώσες μετατόπισης και στροφής. Η επίδραση της διατμητικής υστέρησης διερευνάται μέσα από ένα αριθμητικό παράδειγμα με πρακτικό ενδιαφέρον.