ANALYSIS OF RING BEAMS FOR DISCRETELY SUPPORTED CYLINDRICAL SILOS

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1. ABSTRACT

In cylindrical silos, the presence of discrete supports results in a non-uniformity of meridional stresses around the circumference. In larger silos, a sizeable ring beam is normally used, resting on discrete column supports beneath the cylindrical shell to redistribute the majority of the discrete forces from the column supports into a more uniform stress state in the cylindrical wall. Although there is a complex interaction between cylinders and supporting ring beam, EN 1993-4-1 provides the bending and torsional moment equations produced in the isolated ring beam under uniform transverse loading by assuming that the meridional stress distribution in the shell is circumferentially uniform. In this study, these equations given EN1993-4-1 for the design of the ring beam are re-derived using Vlasov's curved beam theory. Then, the shear force equation in the transverse direction for the ring beam is derived using the equilibrium equations. Moreover, the transverse displacement of the ring beam is obtained from the global force-deformation relationships. These closed-form equations are verified by complementary finite element analyses for practical design applications.

2. INTRODUCTION

The cylindrical metal shell is the commonest form for metal silos and tanks. These structures can be supported either on the ground or on a few column supports, depending on the requirements of the discharge system. Columns at equal circumferential spacing are generally employed to elevate the silo structure (*Fig. 1*). The presence of the discrete supports results in a non-uniformity of meridional stresses around the circumference. In lighter silos, the discrete local columns are commonly attached to the side of the wall using either a number of brackets or engaged columns [1]. In larger silos, a sizeable ring beam is normally used, resting on discrete column supports beneath the cylindrical shell. The function of the ring beam is twofold. First, it is required to carry circumferential forces to maintain support for the radial component of the meridional tension in the hopper [2]. Second, the ring beam plays an important role in redistributing majority of the discrete local forces from the column supports into a more uniform stress state in the cylindrical wall [2].

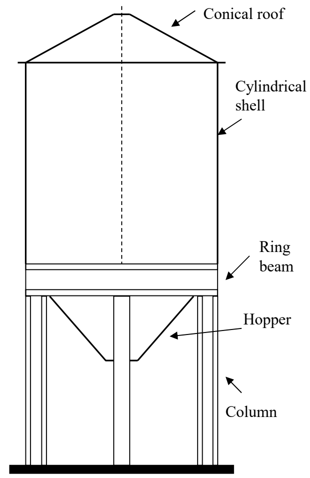
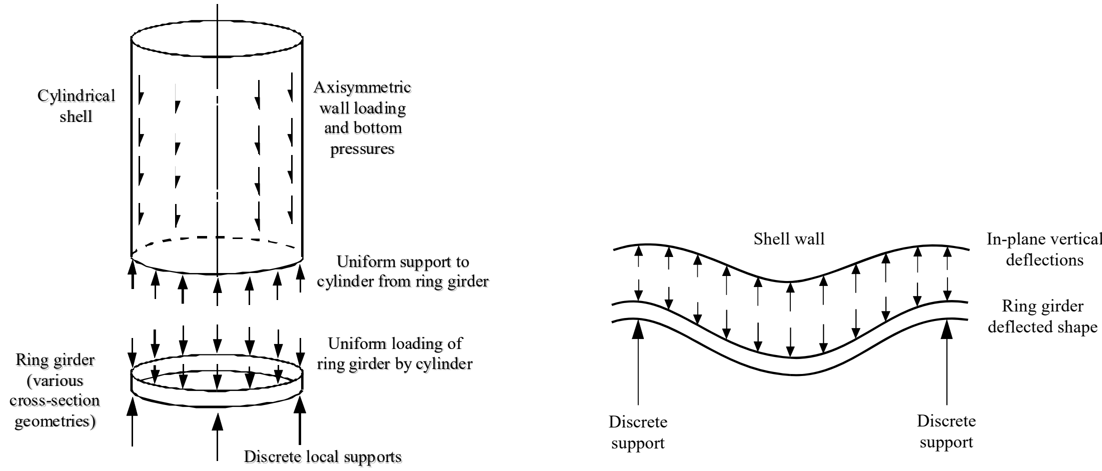


Fig. 1: Typical circular planform silo

Early studies of discretely supported cylinders [3-5] demonstrated the great complexity of this behaviour, and attempts to deal with the interaction of cylinders and supporting ring beams have not led to simple design procedures [6, 7]. Although there is a complex interaction between the cylindrical shell and the ring beam, design is typically greatly simplified by assuming that the meridional stress distribution in the shell is circumferentially uniform. As illustrated in *Fig. 2a*, the tradition is for each component to be treated separately under the action of uniform loading around the circumference [2, 8-10]. For this loading to be valid, the ring beam must function properly in redistributing the discrete support loads into a more uniform state of stress. The extent to which this redistribution of the support forces can be achieved is directly related to the stiffness of the ring beam relative to the stiffness of the cylindrical shell. In other words, deformations of the cylindrical shell and the ring beam must be compatible as shown in *Fig. 2b* for the validity of the underlying assumption. Considering that the cylindrical shell is rather stiff in its own plane, the ring beam should be very stiff, and much stiffer than the shell, to be able to satisfactorily redistribute the support loads. An approximate criterion to determine the appropriate ring beam stiffness was first identified by Rotter [8] and was further developed and verified by Topkaya and Rotter [10]. The criterion developed by these authors is very demanding and usually leads to very big ring beams for typical geometries.
EN 1993-4-1 [9] only provides design equations for stress resultants produced in the isolated ring beam under uniform transverse loading. In this paper, these stress resultants produced in the ring beam were re-derived using Vlasov's curved beam differential equations [11, 12]. Then, the shear force equation in the transverse direction for the ring beam is derived using the equilibrium equations. The advantage of using Vlasov's equations is that the transverse displacements can also be obtained from the differential relationships. Then aforementioned design equations for cases where the ring beam

interacts with the silo shell are verified by complementary finite element analyses for practical design applications.



a) Traditional design model for discretely supported silos

b) Deformation requirement on cylinder imposed by compatibility with beam deformation

Fig. 2: Axial deformation compatibility between ring girder and shell [2, 8, 9]

3. ALGEBRAIC CLOSED-FORM SOLUTION OF STRESS RESULTANTS IN THE RING BEAM USING VLASOV'S CURVED BEAM THEORY

The six basic equilibrium equations for the curved beam element shown in *Fig. 3* can be expressed using the Vlasov's differential equations as follows [11-12]:

$$\frac{1}{r_g}\left[\frac{dQ_r}{d\theta}+Q_\theta\right]+q_r=0 \qquad \frac{1}{r_g}\frac{dQ_x}{d\theta}+q_x=0 \qquad \frac{1}{r_g}\left[\frac{dQ_\theta}{d\theta}-Q_r\right]+q_\theta=0 \tag{1}$$

$$\frac{1}{r_g}\left[\frac{dM_r}{d\theta}+T_\theta\right]-Q_x+m_r=0 \qquad \frac{1}{r_g}\frac{dM_x}{d\theta}+m_x+Q_r=0 \qquad \frac{1}{r_g}\left[\frac{dT_\theta}{d\theta}-M_r\right]+m_\theta=0 \tag{2}$$

where M_r = bending moment in the ring about a radial axis; M_x = bending moment in the ring about a transverse axis; T_θ = torsional moment in the ring; r_g = radius of the ring beam centroid; θ = circumferential coordinate; q_x, q_θ, q_r = distributed line loads per unit length in the transverse, circumferential and radial directions respectively; m_x, m_θ, m_r = distributed applied torques per unit circumference about the transverse, circumferential and radial directions respectively; Q_θ = circumferential force in the ring; Q_x, Q_r = shear forces in the ring in transverse and radial directions respectively.

The free body diagram of a closed section ring beam cross-section is indicated in *Fig. 4*. By examination of this free body diagram, the forces acting in the two principal directions, the uniformly distributed vertical load (n_v) and the radial inward load (n_r), can be expressed as follows:

$$n_v = n_{xc} + n_{\phi h}\cos\beta \qquad n_r = n_{\phi h}\sin\beta \tag{3}$$

where n_{xc} = the design value of compressive membrane stress resultant at the base of the cylinder; $n_{\phi h}$ = the design value of tensile membrane stress resultant at the top of the hopper; β = the hopper half angle.

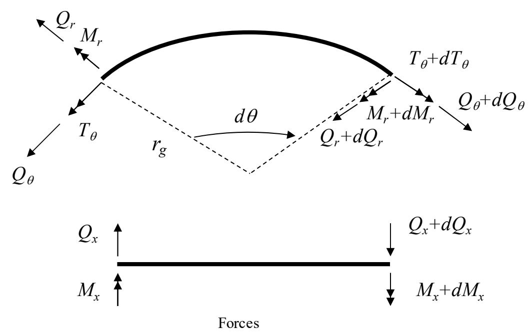


Fig. 3: Differential curved beam element and sign conventions

As shown in *Fig. 4* the forces in the two principal directions act eccentrically with respect to the ring beam centroid (*G*). These forces can be decomposed into three basic loads on the ring beam shown in *Fig. 4* where the vertical and inward loads act through the centroid of the cross-section. The three basic loads are the transverse distributed load (q_x), the circumferentially distributed torque (m_θ), and the concentrated torque at the supports (m_s) which can be expressed by the following relationships:

$$q_x = \frac{n_v(r_g\text{-}e_r)}{r_g} \qquad m_\theta = \frac{n_v(r_g\text{-}e_r)}{r_g}e_r + \frac{n_r(r_g\text{-}e_r)}{r_g}e_x \qquad m_s = Qe_s = 2n_v(r_g\text{-}e_r)\theta_0 e_s \tag{4}$$

where e_r = the radial eccentricity of the cylinder from the ring beam centroid; e_s = the radial eccentricity of the support from the ring beam centroid; e_x = the vertical eccentricity of the joint centre from the ring beam centroid; θ_0 = the circumferential angle in radians subtended by the half span of the ring beam ($\theta_0 = \pi/n$); n = the number of equally spaced discrete supports; Q = support force.
The ring beam is analyzed for these three different load cases in turn.

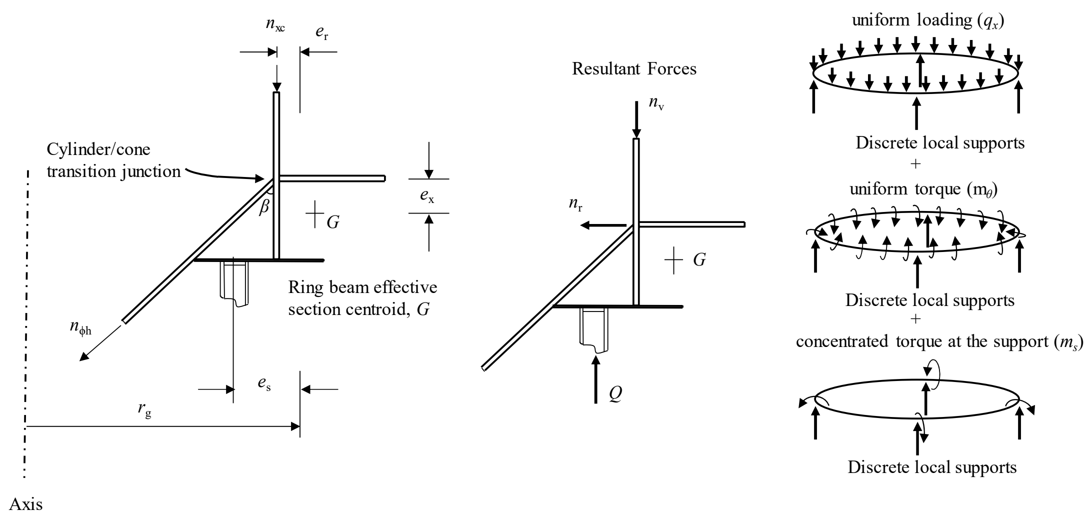


Fig. 4: Simplified load-carrying mechanism model for the ring beam [2]

3.1 Derivation of stress resultants – ring under transverse distributed load

For the case where only a transverse distributed load (q_x) acts on the ring beam (i.e. $q_r = q_\theta = m_r = m_\theta = m_x = 0$), the six basic equilibrium equations can be reduced to three differential relationships. One differential equation concerns bending of the ring in its own plane and can be uncoupled from the other two. The two coupled differential equations of equilibrium can be expressed as :

$$\frac{1}{r_g}\left[\frac{dT_\theta}{d\theta}-M_r\right]=0 \qquad \frac{1}{r_g^2}\left[\frac{d^2M_r}{d\theta^2}+\frac{dT_\theta}{d\theta}\right]=-q_x \tag{5}$$

Simultaneous solution of eq. (5) using the appropriate boundary conditions $M_r(0)=M_r(2\theta_0)$, $T_\theta(0)=T_\theta(2\theta_0)=0$) reveals the following relationships:

$$M_r(\theta)=n_v r_g(r_g-e_r)\left[\theta_0(\sin\theta+\cot\theta_0\cos\theta)-1\right] \tag{6}$$

$$T_\theta(\theta)=n_v r_g(r_g-e_r)\left[\theta_0(\cot\theta_0\sin\theta-\cos\theta+1)-\theta\right] \tag{7}$$

3.2 Derivation of stress resultants – ring under circumferentially distributed torque

In the same manner, the following two coupled differential equations are obtained for the case where only a circumferentially distributed torque (m_θ) is applied to the ring (i.e. $q_x = q_r = q_\theta = m_r = m_x = 0$):

$$\frac{1}{r_g}\left[\frac{dT_\theta}{d\theta}-M_r\right]=-m_\theta \qquad \frac{1}{r_g^2}\left[\frac{d^2M_r}{d\theta^2}+\frac{dT_\theta}{d\theta}\right]=0 \tag{8}$$

Simultaneous solution of eq. (8) using the appropriate boundary conditions $M_r(0)=M_r(2\theta_0)$ and $T_\theta(0)=T_\theta(2\theta_0)=0$) reveals the following relationships:

$$M_r(\theta)=(r_g-e_r)(n_v e_r+n_r e_x) \qquad T_\theta(\theta)=0 \tag{9}$$

3.3 Derivation of stress resultants – ring under concentrated torque at the supports

The following coupled differential equations are obtained for the case where the concentrated torques (m_s) are applied at the supports of the ring (i.e. $q_x = q_r = q_\theta = m_x = m_r = m_\theta = 0$) :

$$\frac{1}{r_g}\left[\frac{dT_\theta}{d\theta}-M_r\right]=0 \qquad \frac{1}{r_g^2}\left[\frac{d^2M_r}{d\theta^2}+\frac{dT_\theta}{d\theta}\right]=0 \tag{10}$$

Simultaneous solution of eq. (10) using the appropriate symmetry boundary conditions that consider symmetry ($M_r(0)=M_r(2\theta_0)$, $T_\theta(\theta_0)=0$ and $T_\theta(0)=m_s/2$) reveals the following relationships:

$$M_r(\theta)=-n_v e_s(r_g-e_r)\theta_0(\sin\theta+\cot\theta_0\cos\theta) \tag{11}$$

$$T_\theta(\theta)=n_v e_s(r_g-e_r)\theta_0(\cos\theta-\cot\theta_0\sin\theta) \tag{12}$$

Eqs (6), (9) and (11) can be superposed to obtain the bending stress resultant and similarly Eqs (7), (9) and (12) can be superposed to obtain the torsional stress resultant as follows :

$$M_r(\theta)=n_v(r_g-e_r)\left[(r_g-e_s)\theta_0(\sin\theta+\cot\theta_0\cos\theta)-r_g+e_r\right]+n_r e_x(r_g-e_r) \tag{13}$$

$$T_\theta(\theta)=n_v(r_g-e_r)\left[(r_g-e_s)\theta_0(\cot\theta_0\sin\theta-\cos\theta)+r_g(\theta_0-\theta)\right] \tag{14}$$

Eqs (13) and (14) are identical to those ones developed by Rotter [2] and provided in EN 1993-4-1 [9].

Shear force in transverse direction can be obtained using eq. (2) as follows:

$$Q_x(\theta)=n_v(e_r-r_g)(\theta-\theta_0) \tag{15}$$

One advantage of using the Vlasov's curved beam differential equations is that the transverse displacements of the girder can be obtained. The global force-deformation relationships for the transverse displacements can be expressed as follows [12]:

$$M_r=\frac{-EI_r}{r_g}\left(\frac{d^2u_x}{r_g d\theta^2}-\phi\right) \qquad T_\theta=\frac{GK_T}{r_g}\left(\frac{d\phi}{d\theta}+\frac{1}{r_g}\frac{du_x}{d\theta}\right)-\frac{EI_w}{r_g^3}\left(\frac{d^3\phi}{d\theta^3}+\frac{1}{r_g}\frac{d^3u_x}{d\theta^3}\right) \tag{16}$$

where u_x = transverse (vertical) displacement normal to the plane of the ring, as shown in *Fig. 3*; ϕ = rotation; I_r = bending moment of inertia of the ring beam about radial axis; I_w = warping constant of the ring beam; K_T = uniform torsional constant of the ring beam; A = cross sectional area of the ring beam; E = modulus of elasticity and G = shear modulus. The first term in torsional moment expression in eq. (16) represents the response under St. Venant torsion, while the second term represents warping contribution. The present study focuses on closed sections where St. Venant's term dominates over the warping term. Warping is neglected in these calculations.
The resulting bending moment and torsional moment variations which are given in eqs (13) and (14) can be directly inserted eq. (16) to obtain the following differential relationship:

$$\frac{d^3 u_x}{d\theta^3} + \frac{du_x}{d\theta} = r_g^2 \left[\frac{T_\theta}{GK_T} - \frac{1}{EI_r} \frac{dM_r}{d\theta} \right] \tag{17}$$

Eq. (17) can be solved using the appropriate boundary conditions ($u_x(0) = u_x(2\theta_0) = 0$, $u_x'(0) = u_x'(2\theta_0)$) to find the transverse displacements as:

$$u_x(\theta) = \frac{-n_v (r_g - e_r) r_g^2}{4EI_r GK_T \sin(\theta_0)^2} \left\{ \begin{array}{l} EI_r r_g \theta(\theta - 2\theta_0)(1 - \cos 2\theta_0) - (EI_r + GK_T)(r_g - e_s)\theta_0 \left[2\theta_0 + (\theta - 2\theta_0)\cos\theta - \theta\cos(\theta - 2\theta_0) \right] \\ + \theta_0 \left[\sin\theta - \sin(\theta - 2\theta_0) - \sin 2\theta_0 \right] + \left[EI_r (3r_g - e_s) + GK_T (r_g - e_s) \right] \end{array} \right\} \tag{18}$$

4. COMPUTATIONAL VERIFICATION OF THE CLOSED FORM SOLUTIONS

The commercial finite element program, ANSYS v12.1 [13], was used to verify the accuracy of these equations for the stress resultants and displacements in the transverse direction. The isolated ring beam analyzed here rests on n = 4 supports and has a cylinder radius of r_g = 3000 mm. A constant uniform transverse loading (n_v) of 1.5 kN/mm was applied to the ring beam. All eccentricity terms were neglected for simplicity. The ring beam was selected a square hollow section and modelled using two-node beam elements (beam4) as shown *Fig. 5*. The modulus of elasticity was taken as 200 GPa and Poisson's ratio as 0.30.

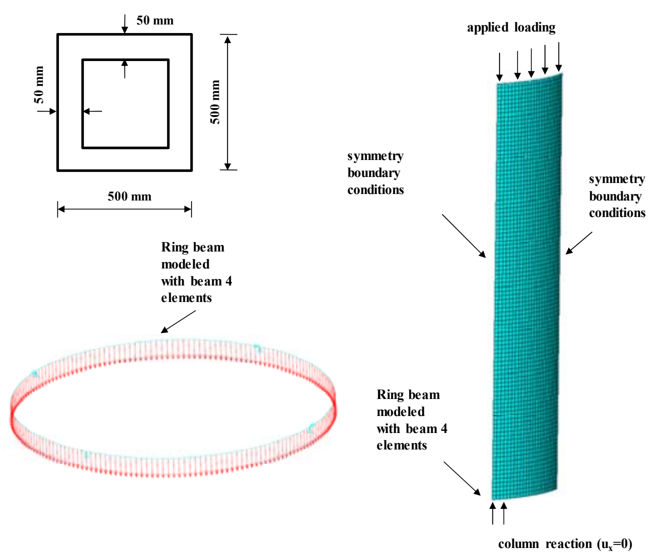


Fig. 5: Finite-element modelling for the ring beam and cylindrical shell

The calculated variations of M_r, T_θ, Q_x and u_x in the circumferential direction are shown in *Fig. 6*, together with the predictions of the closed form solutions (eqs (13), (14), (15) and

(18)). When an isolated ring beam is considered, the comparisons show that the above equations provide very accurate solutions, with the largest differences being 0.18%, 0.26%, 4.82% and 0.13% for M_r, T_θ, Q_x and u_x respectively.
To investigate the behaviour of a ring that interacts with the silo shell, silo structures with a ring beam were next considered with different shell thicknesses t = 4 and 10 mm and a constant height H = 10000 mm. The same support conditions and ring beam properties were used. A uniform axial load (n_v = 1.5 kN/mm) was applied to the top of the cylindrical shell around the full circumference. For a cylindrical shell resting on n equally spaced discrete supports, there are $2n$ planes of symmetry. The computational time was reduced by modelling only a segment covering an angle of θ/n, as shown in *Fig. 5*. Four-node shell elements (shell63), with a size of 100 mm in both the axial and circumferential directions, were employed to model the cylindrical shell.

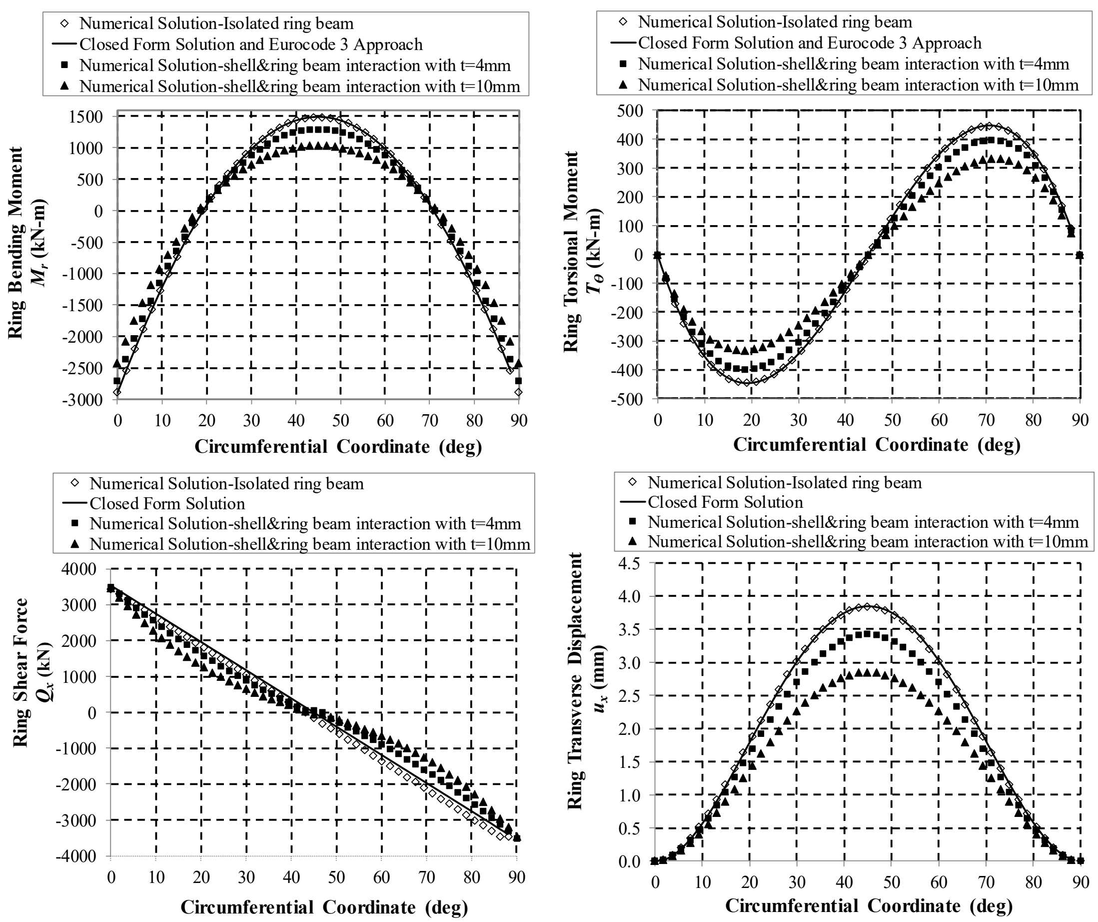


Fig. 6: Comparison of closed form solution with numerical solutions for the ring that interacts with the silo shell

The variation of the stress resultants and displacement are shown in *Fig. 6*. When the 4 mm thick shell was considered, the comparisons show that the above equations provide conservative solutions for the ring, with the largest differences being 15.54%, 13.02%, 22.83% and 12.21% for M_r, T_θ, Q_x and u_x respectively. When the 10 mm thick shell was considered, the comparisons show that the above equations provide more conservative solutions for the ring, with the largest differences being 44.52%, 36.80%, 58.49% and 34.82% for M_r, T_θ, Q_x and u_x respectively.

5. SUMMARY AND CONCLUSIONS

This paper has both developed and re-derived design equations for ring beams used to support cylindrical shells. The study concentrated on closed section ring beams where the warping term is neglected in the calculations. Closed form design equations obtained from Vlasov's curved beam theory were compared with numerical results. The comparisons show that these equations provide acceptably accurate solutions when the isolated ring beam is considered. On the other hand, when the ring beam and shell interaction is considered, the values obtained from finite element calculations diverge from those of the closed form solutions. As the thickness of the shell gets thicker, the difference between the closed form solution and the numerical calculations increases. The design of a ring beam as an isolated curved beam is conservative, since it neglects the contribution of the attached shell and hopper.

6. REFERENCES

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