# A 2D FRAME ELEMENT TAKING INTO ACCOUNT SEMI-RIGIDITY AT COLUMN BASES, BRACE ENDS & BEAM TO COLUMN CONNECTIONS IN STEEL STRUCTURES

**Halil Fırat Özel**
Research Assistant, M.S., Ph.D. Candidate
Middle East Technical University
Ankara, Turkey
E-mail: fozel@metu.edu.tr

**Afsin Saritas**
Professor, Ph.D.
Middle East Technical University
Ankara, Turkey
E-mail: asaritas@metu.edu.tr

## 1. ABSTRACT

In this paper, a force-based 2D frame element formulation with spread of inelasticity through the element and localized semi-rigid connections at beam, brace and column base regions is presented. Inelasticity along member lengths is captured through fiber discretization of monitoring sections; furthermore, defining any type of semi-rigid connection along element length does not require introduction of additional nodes and degrees of freedom to the system. An accurate shear correction coefficient for wide flange sections is also taken into account in order to get closer match of shear effects with respect to exact solutions. The proposed element is designated for the performance based analysis of steel structures, since considering the presence of semi-rigid connections at beam column, column base and brace end regions through analyses will provide more realistic representation of the actual behavior of a structure. The proposed model is tested through available analysis program for validation purpose of gusset plate flexibility on brace ends in frames and the effect of the semi-rigidity on column bases with altering beam to column connection flexibility.

Keywords: Semi-rigid connection; Column base connection; Beam to column connection; Brace end (gusset plate) connection ; Steel framed structures

## 2. INTRODUCTION

Connections in steel structures are defined in two categories to simplify design and analysis phases. The behavior of steel connections is considered as either simple (shear) or moment type (fixed). Since the expected behavior of a connection should have relative rotation together with moment transfer which is called semi-rigid connection, i.e. the real response of the connection is actually partially-restrained. In order to present the actual modal behavior of steel structures, many researchers tend to include the real behavior of connections on the steel structures. Laboratory test on steel connections were conducted by Chui and Chan [1] and Nader and Astaneh-Asl [2] and experimental results were accompanied with numerical analyses. In order to gain better match, numerical analyses considered the real behavior of connections, and much better response was captured. Galvo et al. [3], Siva et al. [4], Al-Aasam and Mandal [5] have developed FEM formulations to examine the dynamic behavior of steel frames with partially-restrained connections. Literature reveals that proper modeling of the behavior of connections have vital role on the dynamic behavior of steel framed structures. Structural design codes offer the influence of structures under dynamic response. The study by Özel, Saritas and Tasbahji [6] presented the necessity of defining connection behavior for accurate modelling of vibration characteristics. Column base connections also necessitate similar tendency on connection behavior. Studies reveal the column base connections show flexible joint behavior [7-10]. Under monotonic loading, Abdollahzadeh and Ghobadi [11], presented the comparison of column base with experimental, analytical and FEM model that was proposed by [12]. The simplified mathematical formulations for column base plates were studied by Stamatapoulos and Ermopoulos [12] to present the flexible joint behavior of column bases under cyclic loading. Another element of the steel framed structures is the brace elements. To complete a system, it is necessary to mention these members. The importance of behavior of brace end connections for steel framed structures is presented in [13, 14]. Most of the cases for the gusset plates are the axial load deformation relation [15-17] and the flexibility of axial direction should be considered.

In order to accomplish an accurate dynamic analysis of steel framed structures, vibration characteristics of typical steel beams, braces and columns with partially restrained connections or also called as in most cases as semi-rigid connections should be studied. For an accurate representation of the semi-rigidity, behavior of the members should be validated. In this work, a mixed formulation frame finite element is implemented with the use of three fields Hu-Washizu-Barr functional. Moreover, an accurate shear area correction of the members is also necessary to validate the actual frame behavior of the steel members. A shear correction factor for I-shaped beams presented by Charney et al. [18] is adopted. The determination of vibration frequencies of members is verified with proposed model that have ability to present altering geometry and material distribution with semi-rigid connections at any section on the element without further specification of different displacement shape functions. SAP2000 [19] is used to validate the results of the proposed model.

## 3. FRAME ELEMENT FORMULATION

### 3.1 Kinematic Relations

Normal and shear strains for a beam section deforming in xy-plane can be written as follows:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_a(x) - y\kappa(x) \\ \gamma(x) \end{Bmatrix} = \mathbf{a}_s(y,z)\,\mathbf{e}(x) \tag{1}$$

where $\mathbf{e}(x)$ is the section deformation vector given as follows;

$$\mathbf{e}(x) = \begin{bmatrix} \varepsilon_a(x) & \gamma(x) & \kappa(x) \end{bmatrix}^{\mathrm{T}} \tag{2}$$

where $\varepsilon_a(x)$ is the axial strain of the reference beam axis, $\gamma(x)$ is the shear deformation of the section along y-axis and $\kappa$ is the curvature of the section about z-axis. Finally, section compatibility matrix $\mathbf{a}_s$ in Equation (1) is written as follows;

$$\mathbf{a}_s(y,z) = \begin{bmatrix} 1 & 0 & -y \\ 0 & 1 & 0 \end{bmatrix} \tag{3}$$

### 3.2 Basic System without Rigid Body Modes and Force Interpolation Functions

The cantilever basic system shown in Figure 1 is used for removing the rigid body modes in the element response. This basic system provides easier derivation of the mass matrix of the beam than the use of simply supported basic system.

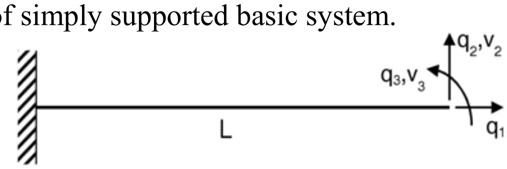


Figure 1. Cantilever basic system forces and deformations

Element formulation is given in xy-plane, where the formulation presents two end nodes and is based on a transformation from complete system to basic system. In the structure of the formulation, the element has 3 degrees of freedom (dof) per node, resulting in 6 dofs, where the nodes are placed at element ends. The complete system is offered such that the axis of the element is aligned with horizontal x-axis. The transformation matrix, **a** for an element with length L is used to relate element end forces and deformations in complete system to basic element forces as follows;

$$\mathbf{p} = \mathbf{a}^{\mathrm{T}}\mathbf{q} \quad \text{and } \mathbf{v} = \mathbf{a}\,\mathbf{u}; \quad \text{where} \quad \mathbf{a} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -L & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Basic element forces at free end, $\mathbf{q}$ are shown in Figure 1 and given in Equation (4). These forces can be related to internal section forces, $\mathbf{s}(x)$ by using the force interpolation matrix $\mathbf{b}(x,L)$ for the cantilever beam configuration as follows;

$$\mathbf{s}(x) = \begin{bmatrix} N(x) & V(x) & M(x) \end{bmatrix}^{\mathrm{T}} = \mathbf{b}(x,L)\mathbf{q} + \mathbf{s}_{\boldsymbol{p}}(x)$$

$$\mathbf{b}(x,L) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & (L-x) & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{s}_{\boldsymbol{p}}(x) = \begin{bmatrix} L-x & 0 \\ 0 & L-x \\ 0 & (L-x)^2/2 \end{bmatrix} \begin{Bmatrix} w_x \\ w_y \end{Bmatrix} \tag{5}$$

**3.3 Variational Base and Finite Element Formulation of the Element**

Variational form of the element is written by independent element nodal displacements **u**, element basic forces **q**, and section deformations **e** by using three-fields Hu-Washizu functional and implemented as part of beam finite elements by [20] and [21]. Extension to dynamic case is achieved through introduction of inertial forces $\mathbf{m}\ddot{\mathbf{u}}$ to get the following variational form of the element

$$\begin{aligned}\delta\Pi_{\mathrm{HW}} &= \int_0^L \delta\mathbf{e}^{\mathrm{T}}\left(\hat{\mathbf{s}}(\mathbf{e}(x)) - \mathbf{b}(x,L)\,\mathbf{q} - \mathbf{s}_{\boldsymbol{p}}(x)\right)dx - \delta\mathbf{q}^{\mathrm{T}}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{e}(x)\,dx + \delta\mathbf{q}^{\mathrm{T}}\mathbf{a}_g\mathbf{u} \\ &\quad + \delta\mathbf{u}^{\mathrm{T}}\mathbf{a}_g{}^{\mathrm{T}}\mathbf{q} + \delta\mathbf{u}^{\mathrm{T}}\mathbf{m}\ddot{\mathbf{u}} - \delta\mathbf{u}^{\mathrm{T}}\mathbf{p}_{app} = 0\end{aligned} \tag{6}$$

To satisfy Equation (6) for arbitrary $\delta\mathbf{u}$, $\delta\mathbf{q}$ and $\delta\mathbf{e}$, we get

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{p} \equiv \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{p} = \mathbf{a}_{\mathrm{g}}{}^{\mathrm{T}}\mathbf{q} \tag{7}$$

$$\mathbf{v} \equiv \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\,\mathbf{e}(x)\,dx; \quad \text{where} \quad \mathbf{v} = \mathbf{a}_{\mathrm{g}}\mathbf{u} \tag{8}$$

$$\hat{\mathbf{s}}(\mathbf{e}(x)) \equiv \mathbf{b}(x,L)\mathbf{q} + \mathbf{s}_{\boldsymbol{p}}(x) \tag{9}$$

Equation (7) is the equation of motion that stands for linear or nonlinear material response, and this equation can be taken for each element to get structure's equation of motion. A numerical time integration scheme can be used to get a solution.
For linear elastic material response, section deformations can be calculated as $\mathbf{e}=\mathbf{k}_s^{-1}\hat{\mathbf{s}}$ to obtain the section deformations from section forces through the use of section stiffness matrix $\mathbf{k}_s$. The change of section deformations $\mathbf{e}$ to Equation (8) gives:

$$\mathbf{a}_{\mathrm{g}}\mathbf{u} = \mathbf{v} = \mathbf{f}\,\mathbf{q}; \quad \text{where} \quad \mathbf{f} = \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{f}_s(x)\,\mathbf{b}(x,L)\,dx \tag{10}$$

In above equation f is the flexibility matrix of the element in the basic system. $f_s$ is the section flexibility matrix that can be calculated from the inversion of the section stiffness matrix $k_s$. Further substitution of above equation for linear elastic response in Equation (7) results in

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{k} = \mathbf{a}^{\mathrm{T}}\mathbf{f}^{-1}\mathbf{a} \tag{11}$$

where k is the 6×6 element stiffness matrix in the complete system. Mass matrix of the force-based element is written in a 6×6 dimension by the method provided by [22], and the details can be seen in [6, 23]. With this approach, the need to derive displacement shape functions along element length is circumvented.
The presence of semi-rigid connections is now introduced through the following extended version of above equation for the calculation of element end deformations:

$$\begin{aligned}&\mathbf{v} = \mathbf{v}_{\mathrm{Frame}} + \mathbf{v}_{\mathrm{Con}}; \quad \text{where} \quad \mathbf{v}_{\mathrm{Frame}} = \int_L \mathbf{b}^{\mathrm{T}}(x)\mathbf{e}(x)\,dx; \quad \mathbf{v}_{\mathrm{Con}} = \sum_{i=1}^{nSC}\mathbf{b}^{\mathrm{T}}(x_i)\boldsymbol{\Delta}_{SC,i} \\ &\text{and} \quad \boldsymbol{\Delta}_{SC} = \left[\delta_{SC}^{axial} \quad \theta_{SC} \quad \delta_{SC}^{shear}\right]^{\mathrm{T}}\end{aligned} \tag{12}$$

The first integral along the length of the frame element can be numerically calculated by using a quadrature rule to capture spread of inelastic behavior and $n_{SC}$ is the total number of semi-rigid connections discreetly located along element length; $\Delta_{SC}$ is the vector of semi-rigid connection deformations. Introduction of semi-rigid connections along element length in Figure 1 does not alter the force field under small deformations. Element flexibility matrix is similarly discretized as follows:

$$\mathbf{f} = \mathbf{f}_{\mathrm{Frame}} + \mathbf{f}_{\mathrm{Con}}; \quad \text{where} \quad \mathbf{f}_{\mathrm{Frame}} = \int_L \mathbf{b}^{\mathrm{T}}(x)\mathbf{f}_s(x)\mathbf{b}(x)\,dx; \quad \text{and} \quad \mathbf{f}_{\mathrm{Con}} = \sum_{i=1}^{nSC}\mathbf{b}^{\mathrm{T}}(x_i)\mathbf{f}_{SC,i}\mathbf{b}(x_i) \tag{13}$$

### 3.4 Section Response

Section response can be got by the basic assumption that plane sections before deformation remain plane after deformation along the length of the beam by the use of following section compatibility matrix as given in Equation (1), where the section compatibility matrix now contains the shear correction factor $\kappa_s$ as follows

$$\mathbf{a}_{\mathrm{s}} = \mathbf{a}_{\mathrm{s}}(y) = \begin{bmatrix} 1 & 0 & -y \\ 0 & \sqrt{\kappa_s} & 0 \end{bmatrix} \tag{14}$$

Shear correction factor $\kappa_s$ is taken as the inverse of the form factor suggested by Charney et al. [18] for I-section:

$$\kappa_s = 1/\kappa; \quad \text{where} \quad \kappa = 0.85 + 2.32\frac{b_f t_f}{d\, t_w} \tag{15}$$

The section forces are obtained by integration of the stresses that fulfill the material constitutive relations $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon})$ according to

$$\mathbf{s} = \int_A \mathbf{a}_{\mathrm{s}}^{\mathrm{T}} \boldsymbol{\sigma}\, dA; \quad \text{where} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix} \tag{16}$$

The derivative of section forces from (16) with respect to the section deformations results in the section tangent stiffness matrix

$$\mathbf{k}_{\mathrm{s}} = \frac{\partial \mathbf{s}}{\partial \mathbf{e}} = \int_A \mathbf{a}_{\mathrm{s}}^{\mathrm{T}} \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon})}{\partial \mathbf{e}}\, dA = \int_A \mathbf{a}_{\mathrm{s}}^{\mathrm{T}} \mathbf{k}_{\mathrm{m}} \mathbf{a}_{\mathrm{s}}\, dA \tag{17}$$

The material tangent modulus $k_m$ is obtained from the stress-strain relation according to $\mathbf{k}_m = \partial\boldsymbol{\sigma}(\boldsymbol{\varepsilon})/\partial\boldsymbol{\varepsilon}$. Above integrals are numerically calculated, where this approach is named as fiber model.

## 4. NUMERICAL EXAMPLES

For the simulation of the importance of beam-column and column baseplate connection behavior, a 3 bay and 6 stories steel frame shown in Figure 2 is considered. The length of the beams is 6.0 m and the height of the columns is 3.75 m. The columns and beams of the structure consist of HEB260 and IPE300, respectively. Semi-rigid connections are present at both ends of each beam, and as well as column bases. In study [6], the results from proposed model were compared with both SAP2000 analyses and the results given in the study of Al-Aasam and Mandal [5], which is derived for the estimation of the vibrations in semi-rigid steel structures with fixed bases. The study of Özel, Saritas and Tasbahji [6] revealed that flexible joint presents good match with the study presented in [5].

The effect of the column baseplate is introduced in this work and the vibration characteristic of the resulting variation of fundamental vibration frequency is plotted in Figure 3. The effect of the semi-rigid connection at both beam-column joints and column baseplate revealed the drastic change on the dynamic behavior of the steel structure. The blue region on the 3D graph represents the both beam-column and column baseplate connections are closer to pinned case and red region represents connection behaviors that are closer to fixed case. In between these results the change in fundamental natural frequency of the 3 bay 6 story frame with flexible joints is significant.

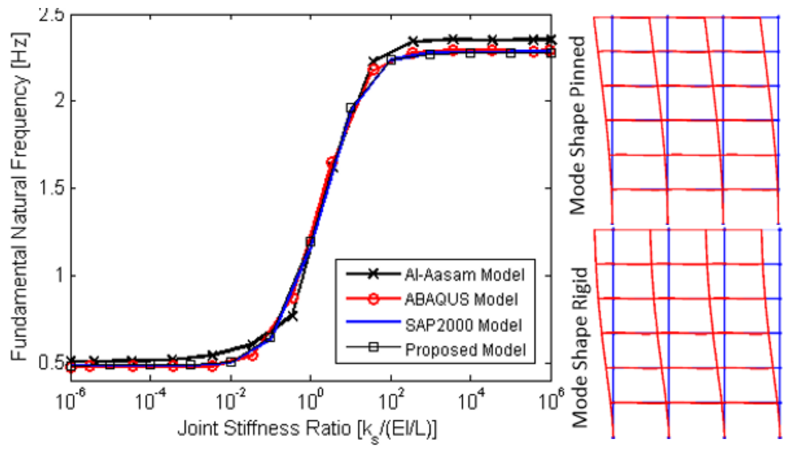


Figure 2. Fundamental Natural Frequency vs. Joint Stiffness Ratio for 3 Bays 6 Stories Steel Framed Structure [6]

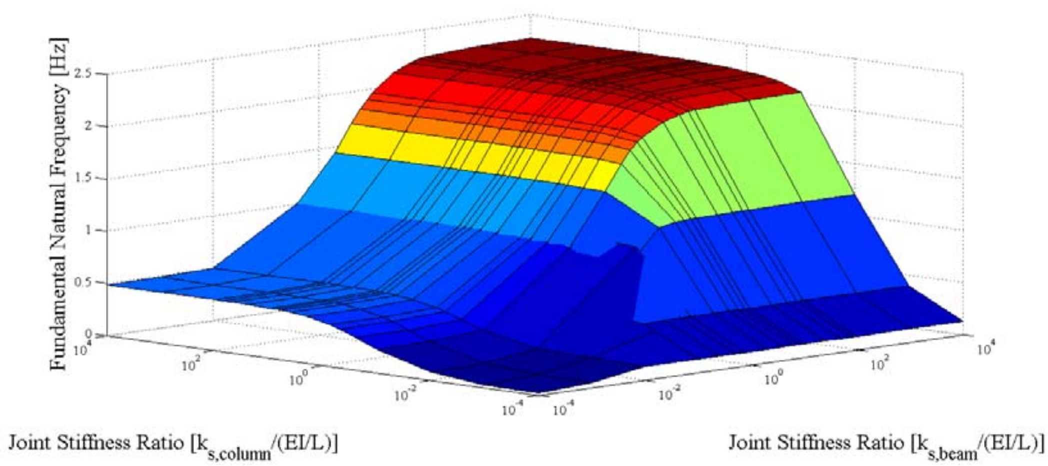


Figure 3. Fundamental Natural Frequency vs. Joint Stiffness Ratio of Beam-column and Column Baseplate Interaction for 3 Bays 6 Stories Steel Framed Structure

In order to study the significance of gusset plate axial flexibility on the vibration characteristic of a structure, the portal frame with brace shown in Figure 4 is considered. This example is also used for the purpose of verification of proposed element's capability to introduce axial stiffness in an inclined element's response, i.e. to the brace member, without the need to introduce new nodes and degrees of freedom to the structural model. The portal frame has 3 m height and 2.9 m span with UC203X203X60 columns, UB254X146X37 beam and RHS70*5 brace. The portal frame is adopted from [5] and brace member is added to observe the effects of flexibility of gusset plates. At the ends of the brace, gusset plates are considered and the flexibility of these gusset plates are denoted with a ratio, $\lambda$ that is obtained by axial rigidity of the brace member divided by the length, EA/L. The range of the $\lambda$ value is plotted in Figure 4 with corresponding normalized fundamental natural frequency of the braced portal frame. Same example is analyzed with SAP2000 and as seen on the Figure 4, very good match between proposed model and SAP2000 model is obtained.

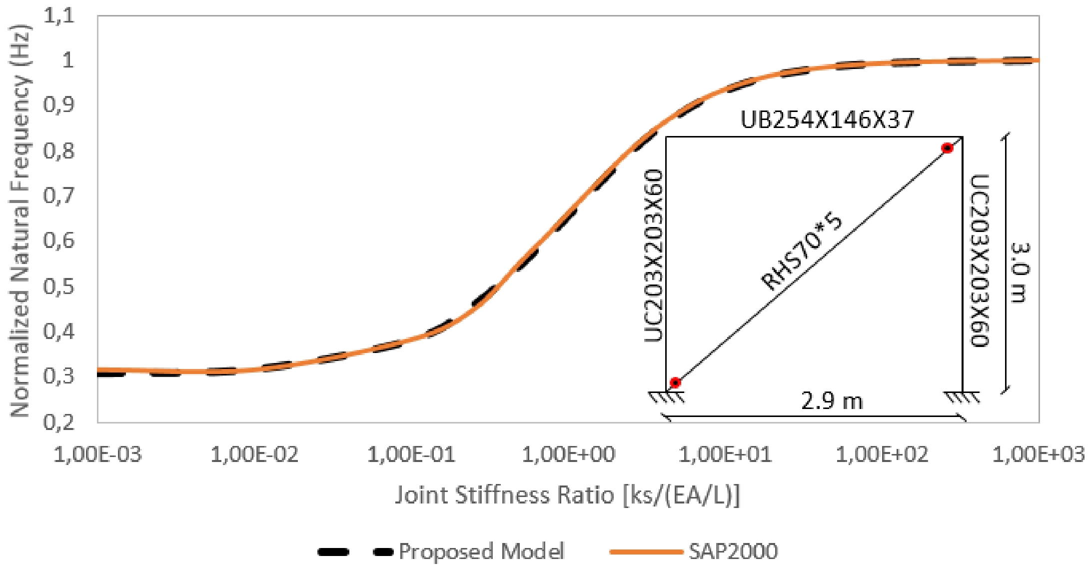


Figure 4. Normalized Natural Frequency vs. Joint Stiffness Ratio for Braced Portal Frame

## 3. CONCLUSIONS

The proposed frame element model can intake any type of localized connection response in a steel framed structure. In this study, the dynamic behavior of beam-column, brace ends and column baseplate connections on steel structures are evaluated with the proposed model. Hence, modal properties for portal braced frame which is made up with rectangular hollow sections and I-sections are accurately captured through comparison with SAP2000 results. Also, the interaction of the flexibility on column bases and beam ends are presented and the change in the dynamic behavior of the 3 bay 6 story frame is observed. The effect of the semi-rigidity on column bases and beam-column connections showed a drastic change on the dynamic behavior of steel structures. Proposed model provides accurate and vigorous solutions for the vibration and dynamic analysis of semi-rigid steel structures by the use of single element discretization per member. This study presented the necessity of the flexible joint definitions at beam, brace and column connections.

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